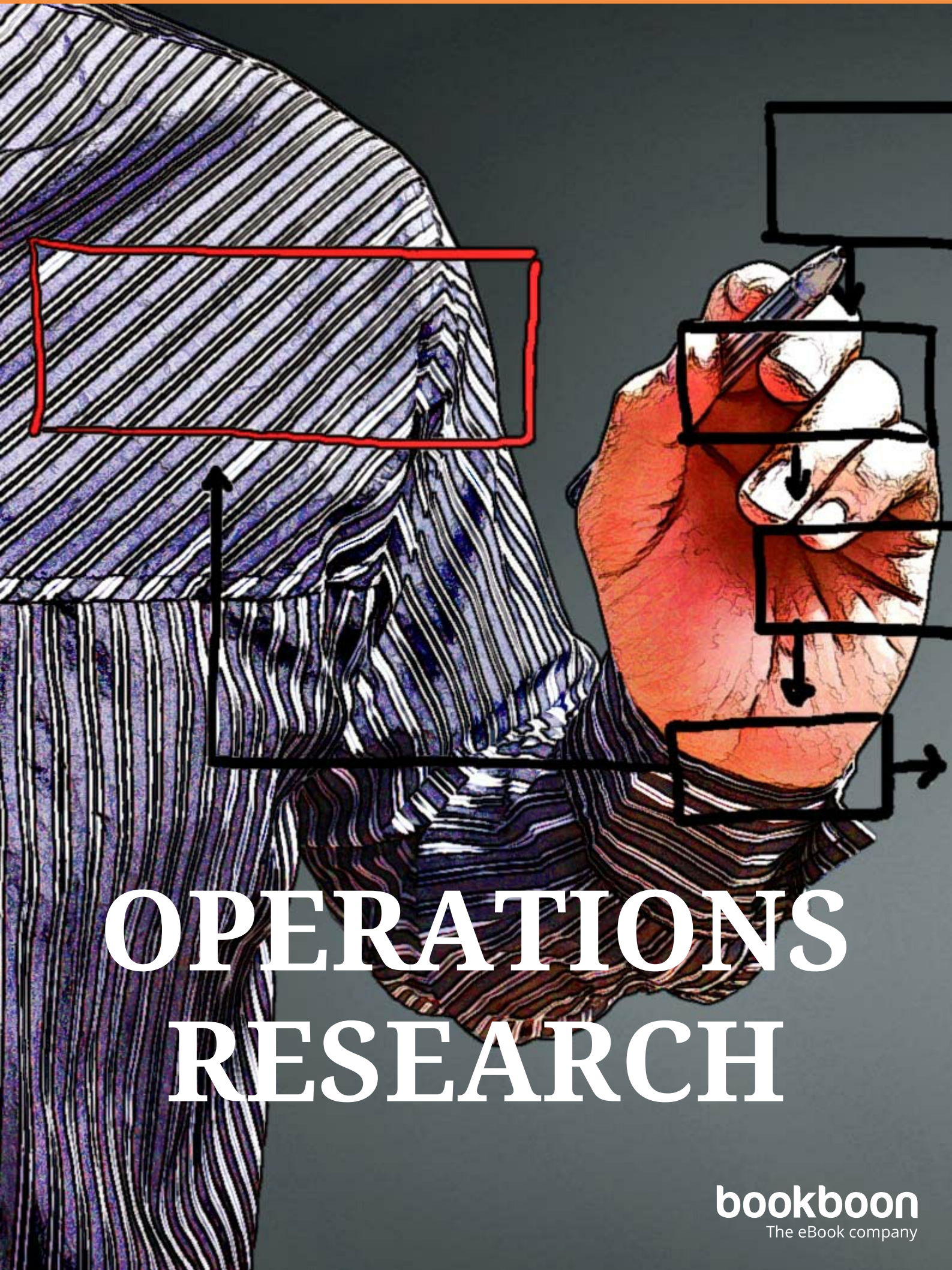


Dr. Yazan K.A-A. Migdadi



OPERATIONS RESEARCH

YAZAN KHALID ABED-ALLAH MIGDADI

OPERATIONS RESEARCH

Operations Research

1st edition

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ABOUT AUTHOR



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PREFACE

This book is an easy guide for the lecturers and students. It helps in choosing the appropriate quantitative analysis techniques, applying the technique, making decision according to outcomes, and conducting the sensitivity analysis. This is an introductory book of operations research covers linear programming graphical solution, simplex method, integer programming, transportation problem, decision making models, and decision tree. At the beginning of each chapter the learning outcomes are listed, and review questions are available at the end of each chapter.

1 INTRODUCTION TO OPERATIONS RESEARCH

1.1 LEARNING OUTCOMES

1. Defining the operations research
2. Contrasting between different terminologies of the field
3. Understanding the revolution of operations research
4. Understanding the operations research approach

1.2 TERMINOLOGIES

The British/Europeans refer to “operational research”, the Americans to “operations research” but both are often shortened to just “OR”.

Another term which is used for this field is “management science” (“MS”). The Americans sometimes combine the terms OR and MS together and say “OR/MS” or “ORMS”. Yet other terms sometimes used are “industrial engineering” (“IE”) and “decision science” (“DS”).

1.3 DEFINITION OF OPERATIONS RESEARCH

Operations Research (OR) is the field of how to form mathematical models of complex management decision problems and how to analyze the models to gain insight about possible solutions.

1.4 WHAT IS OPERATIONS RESEARCH? MANAGEMENT SCIENCE?

Operations research is concerned with scientifically deciding how to best design and operate man-machine systems, usually under conditions requiring the allocation of scarce resources.

Management Science is the application of a scientific approach to solving management problems in order to help managers make better decisions.

OR is the abstraction of real world problems into mathematical models, and the development of techniques to optimally (or attempt to optimally) solve these problems. The application of OR is using the models and methods to aid decision makers

1.5 HISTORY OF OR/MS

It was possible to study physics, mathematics or engineering before 70 years ago, but it was impossible to study operations research. This field is a new one which dates back to late 1930s, it was started in the UK. It was started as the strategic and tactical studying of problems in military operations. The objective was to use to quantitative analysis techniques to find the most effective utilization of military resources.

A.P. Rowe was the first who coined the term operational research to describe the activities of the Air ministry research at Bawadsey in the years between 1937 and 1939. Its mission was related to put radars on operations. During 1930s the UK committee for air defence or what is often called Tizard committee was developed for the purpose to protect the UK air from the coming air attack, this committee formed from three scientists; Tizard the physicist who is the best scientist which the UK ever know, Blackett the awarded of Nobel prize in physics 1948 and, Hill the awarded Noble prize in medicine 1922.

The first operations management groups at USA were developed during the second world war II, these committees were Mine Warfare Operations Research Group, the Anti-Submarine Warfare Operations Research Group, and the Army Air Forces operations analysis sections. The first leading university in teaching operations research was John Hopkins, an office was established for this purpose and was called John Hopkins University Operations Research Office, and the first doctoral program in engineering with a specialty in operations research was offered by John Hopkins university in 1956.

1.6 OPERATIONS RESEARCH SOCIETIES

The International Federation of Operational Research Societies ([IFORS](#)) is an [umbrella organization](#) for operational research societies worldwide, representing approximately 50 national societies including those in the US, UK, France, Germany, Canada, Australia, New Zealand, Philippines, India, Japan and South Africa ([ORSSA](#)): IFORS was born some 20 years later out of professional cooperation. The first International Federation of Operational Research Societies (IFORS) conference, held in Oxford in 1957, was considered by Maurice Kirby as the fifth defining moment in the history of OR. IFORS officially came into existence in January 1959. The constituent members of IFORS form regional groups, such as that in Europe. Other important operational research organizations are [Simulation Interoperability Standards Organization](#) (SISO) and [Interservice/Industry Training, Simulation and Education Conference](#) (I/ITSEC).

The objectives of the Federation shall be to develop operational research as a unified science and to advance it in all the nations of the world. Means to this end shall include:

1. Sponsoring international meetings.
2. Providing other means for exchanging information on operational research.
3. Encouraging nations to establish operational research societies.
4. Supporting standards of competence in operational research.
5. Promoting operational research education.
6. Promoting the growth of both existing and new fields of operational research

1.7 JOURNALS

- Number of journals is 76 journal
- The publisher origins are from 11 countries
- Brazil 1, France 1, Germany 6, India 1, Japan 1, Netherland 14, Singapore 1, Spain 1, Taiwan 1 UK 34, USA 16.
- OR is a new field which started in the late 1930's and has grown and expanded tremendously in the last 30 years. As such the academic journals contain many useful articles that reflect state of the art applications of OR:

Operations Research

Management Science

European Journal of Operational Research

Journal of the Operational Research Society

Mathematical Programming

Networks

Naval Research Logistics

1.8 THE APPROACH OF QUANTITATIVE ANALYSIS

- Problem definition and identification
 - Symptoms and causes of problem.
 - Positive and negative problem
 - example: the increase or decrease of profit is a symptom, however, the causes could be the limitation in resources, the resources prices...etc.
 - objective: increase the profit, the constraints are: the number of resources, prices of resources.
- Formulating the problem by using formulas or models
 - Models are the representation of reality (situation)
 - Models could be mathematical, physical, schematic.
 - The mathematical models contains variables (controllable) and parameters (measurable quantity).

If the values of models are known certainly then models are called deterministic otherwise it is called stochastic (probabilistic) models

e.g. max. profit = $C_1x_1 + C_2x_2$

s.t. $R_1x_1 + R_2x_2 \leq TWH$

$R_2x_1 + R_2x_2 \leq TM$

$x_1, x_2 \geq 0$

- Collecting data

The sources of data are primary and secondary

The analyst should check the reliability and validity of data

Example

profits of previous years

number of produced items of x_1, x_2

total working hours required to produce x_1, x_2

total metal required to produce x_1, x_2 .

- Solving the problem (developing a solution)

Identify the technique (the series of procedures or actions used in solving the problem is called algorithm)

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The technique used is affected by the assumptions.

In previous example if the behaviour of data shows linear relationship with deterministic outcomes of each variable...etc., then Linear Programming (Graphical method) could be adopted

Linear Programming

Non-linear programming

Integer programming

Goal programming

Dynamic programming

Networking

Transportation and assignment model

Inventory control models

Forecasting models

Project management techniques

Markov analysis

Simulation

Queuing and waiting line models

Quality control methods

Decision Making theory

- Testing the solution: Sensitivity analysis; make different recommendations if the conditions affect the problem are changed.
 - What is the range of profit of X1 and X2, what is the range of resources...etc.
- Making decision
 - to realize the maximum profit of both variables it is more beneficial to produce N units of X1 and N units of X2.
 - note: the outcomes not all the time realized as recommended by the

1.9 THE ROLE OF COMPUTER IN QUANTITATIVE ANALYSIS

- Different software was used as LINDO, CAPLEX and MPL.
- The most friendly, easy to use packages available are; POM-QM and Excel QM. In this course these software will be used.
- Link for free download is http://wps.pearsoned.co.uk/ge_taylor_introms_10/150/38494/9854540.cw/content/index.html.

2 LINEAR PROGRAMMING: GRAPHICAL METHOD

2.1 LEARNING OUTCOMES

1. Formulate the problem according to linear programming principles
2. Solve the problem according to linear programming (graphical corner points) principles
3. Make the decision according to linear programming (graphical: corner points) results
4. Conduct the sensitivity analysis for the outcomes of linear programming (graphical: corner points) results


2.2 DEFINITION

Linear programming graphical is transforming the business problem to linear formulas (equations or inequalities) for a problem contains two decision variables in order to reach the optimal solution.

2.3 THE ASSUMPTIONS OF LINEAR PROGRAMMING

1. The number of decision variables is only two:
The number of variables is justified, if the number is more than two It is impossible to presented graphically. The variables should be defined operational, the definition should be measurable and clear. For example if the general motor's operations manager decided to identify the optimal size of production of two classes of vehicles; busses and trucks, the variables are number of busses will be produced and number of trucks will be produces. The number is a measurable unit.
2. The number of objectives is only one:
The linear programming is dealing with one objective, if the objectives are multi; the recommended technique is goal programming. The objective should has an indicator which is called objective indicator, this indicator is a performance indicator, the indicators are numerous as; financial indicators (net profit, return on assets, return on equity, return on investment, total cost...etc.), or non financial as (market share, customers satisfaction, customers loyalty, number of customers will be served, number of audiences...etc.). The objective indicator is expressed in term of maximum or minimum value, as maximize profit, or minimize cost. The objective in general is the value of decision variables which realizes the optimal level of the objective indicator, accordingly, the objective of general motor's corporation manager is to identify the number of busses and trucks will be produced to realize the maximum profit.


3. The contribution rate (coefficient) of each decision variable in realizing the objective should be known. This rate is generated by the linear regression analysis, which is the coefficient of variable. The contribution rate is generated for each unit of the decision variable, and it is the same as the objective indicator. For example: the profit of each bus and the profit of each truck should be known.
4. The realization of each objective is determined by set of constraints: the constraints are limits of the objective. These limits could be resources (human, physical, and financial), primary inputs (product components), demand...etc.
5. The constraints limits should be known. The limits are called the right hand side value; these limits could upper, lower or equal. The upper and lower limits are in the case of inequalities. However, the equal is in the case of equation.
6. The contribution rate (coefficient) of each decision variable in the constraint should be known. This rate is generated by the linear regression analysis, which is the coefficient of variable. The contribution rate is generated for each unit of the decision variable, and it is the same as the constraint type. If it is labour hours, it will be the labour hours required to produce each bus and each truck at general motors.
7. The values of decision variables should be positives or zeros, no negative values. This is an economic logic, it is impossible for a factory to produce -100 units, it is illogical.



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8. The relationship between variables should be linear, this is the most important and a milestone assumption should be examined by the analyst. If this assumption is not available the whole method will not be adopted. The linearity test will be discussed in the next section.

2.4 HOW TO FORMULATE THE PROBLEM

The operations manager at fans factory decided to indentify the size of production of two classes of fans; table and stand in order to maximize the profit, three constraints limits this objective which are; the labour working hours, amount of metal available, and the forecasted demand of both kind of products. The following tables show the facts related to the production process.

Year	Table fans				Table fans			
	Size of production	Total profit	Labour hours	Metal	Size of production	Total profit	Labour hours	Metal
2006	20250	220500	202500	3037.5	21250	221500	212500	2337.5
2007	20560	250600	215680	3598.	21450	251600	235950	2466.75
2008	21520	270150	218826	3830.6	21650	271150	259800	2554.7
2009	22250	275600	219882	4694.8	21850	276600	262200	2796.8
2010	23350	280600	229585	4950.2	22050	281600	298950	3042.9
2011	24522	280600	239950	5247.71	22250	281600	298989	3022.32
2012	24453	280400	239960	5281.85	22450	281400	301528	4243.05
2013	25520	280900	242658	5537840	22650	281900	301529	4348800
2014	26550	281500	245890	5761350	22850	282500	302529	4455750
2015	27888	282600	256890	6107472	23050	283600	303529	4563900

1. the linear relationship should be examined; the relationship could be examined by using R^2 value (coefficient of determination). This value should be reported for the relationship between quantity of production of each kind of products and profits, and quantity of production and labour hours, quantity of production and metal, and time progressing and size of production. (SPSS or Microsoft Excel data analysis application could be used for this purpose.) The following table shows the values of R^2 .

Relationship	R2	F-value	p-value	Linear
The impact of size of produced table fans and profit	0.54	9.67*	0.014	Yes
The impact of size of produced stand fans and profit	0.60	11.94**	0.01	Yes
The impact of size of produced table fans and labour hours	0.85	43.85**	0.0002	Yes
The impact of size of produced stand fans and labour hours	0.80	31.43**	0.0005	Yes
The impact of size of produced table fans and metal	0.93	103.43**	0.000	Yes
The impact of size of produced stand fans and metal	0.91	81.91**	0.000	Yes
The impact of time progressing and size of produced table fans	0.99	536.63**	0.000	Yes
The impact of time progressing and size of produced stand fans	0.99	536.73**	0.000	Yes

- Decision variable are two; number of produced table fans and number of produced stand fans.
- the contribution rate of each product in realizing objective is computed by using linear regression analysis. The following table shows the result of regression analysis for the relationship between size of production and profit.

Products	Coefficient (profit per unit)	t-value	P-value
Table fans	5.79 JD.	3.12**	0.01
Stand fans	25.52 JD.	3.46**	0.008

The previous facts could be used to formulate the objective function as following

$$\text{Max. profit} = 5.79 X_1 + 25.52 X_2$$


4. the contribution rate of each product in each constraint is computed by using linear regression analysis. The following table shows the result of regression analysis for the relationship between size of production and labour hour, size of production and metal.

Products	Coefficient	t-value	P-value
Table fans	0.20 labour hour	5.61**	0.0005
Stand fans	0.46 labour hour	6.62**	0.0002
Table fans	0.3 kg metal	10.17**	0.000
Stand fans	0.4 kg metal	9.05**	0.000

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5. constraints limits in each constraint are possible to be computed by using linear regression analysis.

Products	Linear regression equation results	Constraint formulation (using the results of step 4 and 5)
Table fans	Up to 28031 units	$X1 \leq 28031$..demand of table fans
Stand fans	Up to 23250 units	$X2 \leq 23250$..demand of stand fans
Table fans	Up to 6582.2 kg of metal	$0.3 X1 + 0.4X2 \leq 11539$..kg of metal
Stand fans	Up to 4957 kg. of metal	
Table fans	Up to 220000 labour hours	$0.2X1 + 0.46X2 \leq 560000$..labor hours
Stand fans	Up to 340000 labour hours	

The problem could be formulated as a standard form as following

Max. profit= $5.79X1 + 25.52X2$

S.T.

$X1 \leq 28031$..demand of table fans
 $X2 \leq 23250$..demand of stand fans
 $0.3 X1 + 0.4X2 \leq 11539$..kg of metal
 $0.2 X1 + 0.46X2 \leq 560000$..labor hours
 $X1, X2 \geq 0$

2.5 HOW TO REPRESENT THE PROBLEM GRAPHICALLY AND FIND THE OPTIMAL SOLUTION

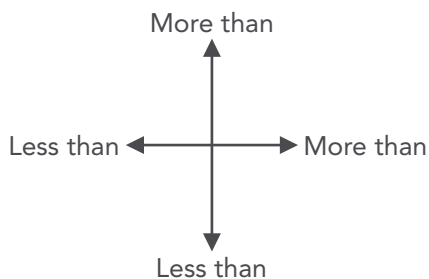
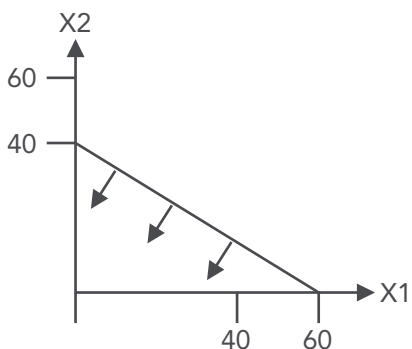
- a) Represents all constraints on the same graph.
- b) Find the feasible region; it a shared region between all constraints.
- c) Identifying the corner points.
- d) Find the gravity of corner points.
- e) Replace the values of corner points on the objective function.
- f) Identifying the point which realizes the optimal level of objective indicators.

2.5.1 REVISION OF CONSTRAINTS REPRESENTATION

1. $2X_1 + 3X_2 \leq 120$

Assume $X_1 = 0 \dots\dots 2(0) + 3X_2 = 120 \dots\dots X_2 = 40$ (0,40)

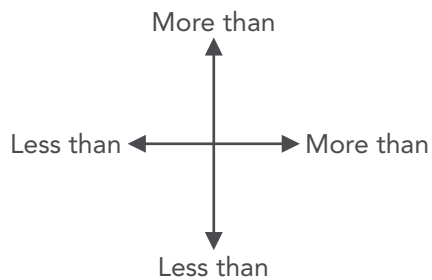
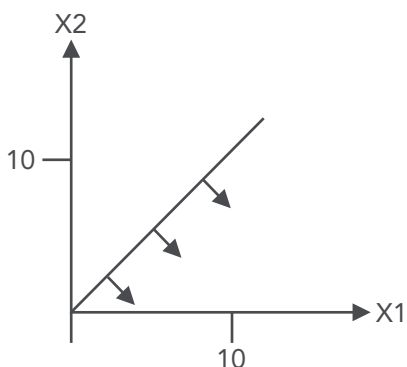
Assume $X_2 = 0 \dots\dots 2X_1 + 3(0) = 120 \dots\dots X_1 = 60$ (60,0)



2. $X_1 \geq X_2$

Assume $X_1 = 0 \dots\dots X_2 = 0$ (0,0)

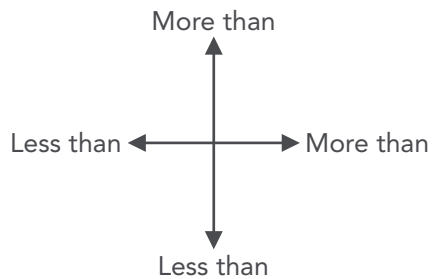
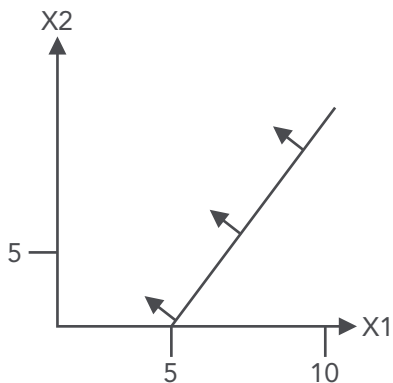
Assume $X_1 = 10 \dots\dots X_2 = 10$ (10,10)



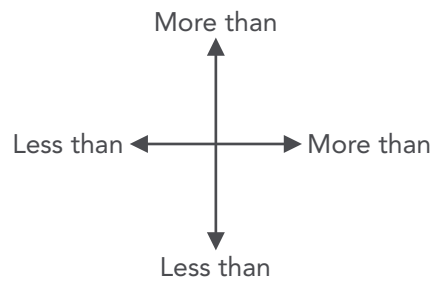
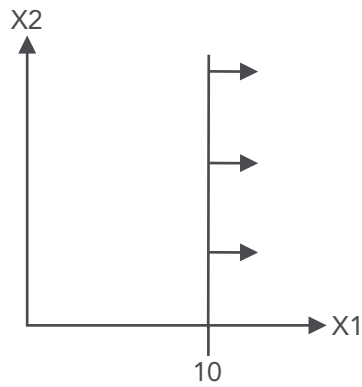
3. $X_1 \leq X_2 + 5$

Assume $X_2 = 0 \dots\dots X_1 = 5$ (5,0)

Assume $X_2 = 5 \dots\dots X_1 = 10$ (10,5)



4. $X_1 \geq 10$



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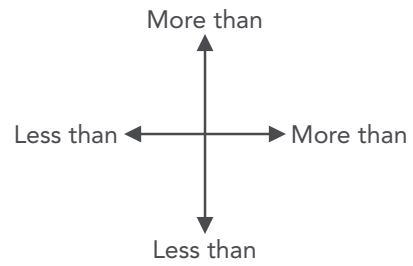
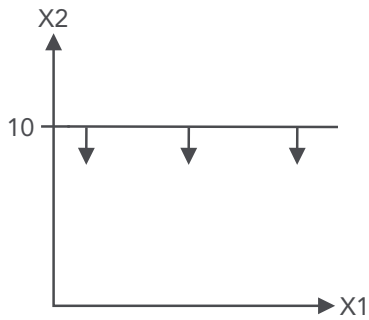
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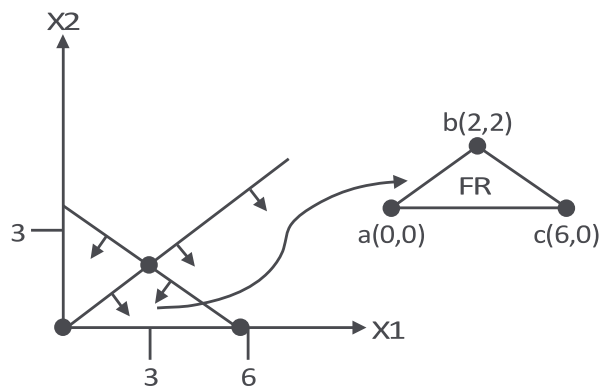
5. $X_2 \leq 10$



2.5.2 GRAPHICAL SOLUTION EXAMPLES

Example 1

Max. profit = $10X_1 + 40X_2$ S.T.
 $2X_1 + 4X_2 \leq 12$
 $X_1 \geq X_2$
 $X_1, X_2 \geq 0$



Corner point	Profit
a (0,0)	0
b (2,2)	100
c (6,0)	60

Optimal solution is producing 2 units of X_1 and 2 units of X_2 , this decision will realize maximum profit of 100 JD.

Example 2

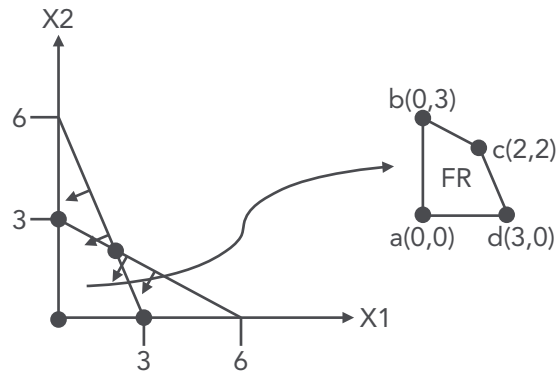
Max. profit = $10X_1 + 40X_2$

S.T.

$2X_1 + 4X_2 \leq 12$

$4X_1 + 2X_2 \leq 12$

$X_1, X_2 \geq 0$



Corner point	Profit
a (0,0)	0
b (0,3)	120
c (2,2)	100
d (3,0)	30

Optimal solution is not producing any unit of X_1 and producing 3 units of X_2 , this decision will realize maximum profit of 120 JD.

Example 3 (special case redundancy)

Max. profit = $10X_1 + 40X_2$

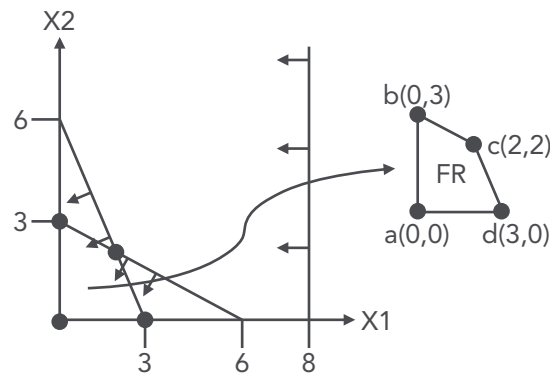
S.T.

$2X_1 + 4X_2 \leq 12$

$4X_1 + 2X_2 \leq 12$

$X_1 \leq 8$...redundant constraint

$X_1, X_2 \geq 0$



Corner point	Profit
a (0,0)	0
b (0,3)	120
c (2,2)	100
d (3,0)	30

Optimal solution is not producing any unit of X1 and producing 3 units of X2, this decision will realize maximum profit of 120 JD.

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Example 4 (special case infeasibility)

Max. profit = $10X_1 + 40X_2$

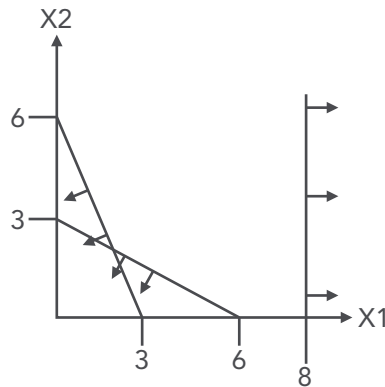
S.T.

$2X_1 + 4X_2 \leq 12$

$4X_1 + 2X_2 \leq 12$

$X_1 \geq 8$

$X_1, X_2 \geq 0$



Example 5 (special case unbounded)

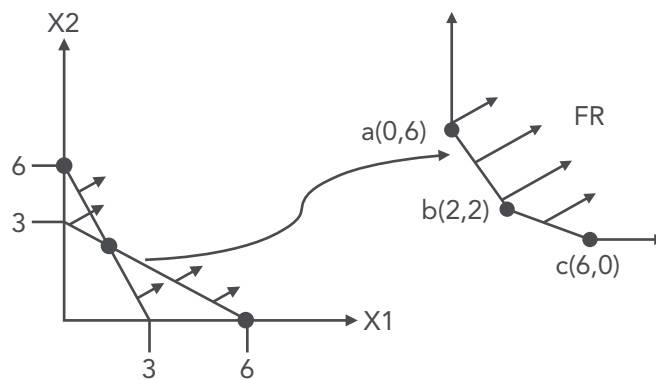
Min. cost = $10X_1 + 40X_2$

S.T.

$2X_1 + 4X_2 \geq 12$

$4X_1 + 2X_2 \geq 12$

$X_1, X_2 \geq 0$



Corner point	cost
a (0,6)	240
b (2,2)	100
c (6,0)	60

Optimal solution is not producing any unit of X2 and producing 6 units of X1, this decision will realize minimum cost of 60 JD.

Example 6 (special case feasible region as a line)

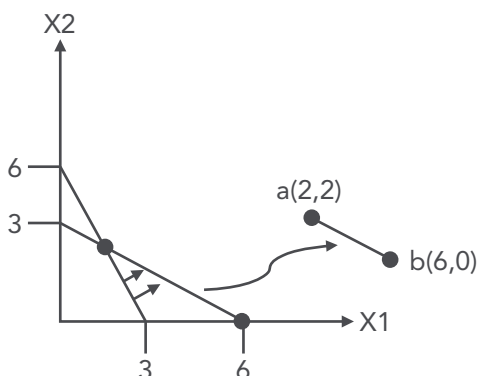
Min. cost = $10X_1 + 40X_2$

S.T.

$2X_1 + 4X_2 = 12$

$4X_1 + 2X_2 \geq 12$

$X_1, X_2 \geq 0$



Corner point	cost
a (2,2)	100
b (6,0)	60

Optimal solution is not producing any unit of X2 and producing 6 units of X1, this decision will realize minimum cost of 60 JD.

Example 7 (special case feasible region as a point)

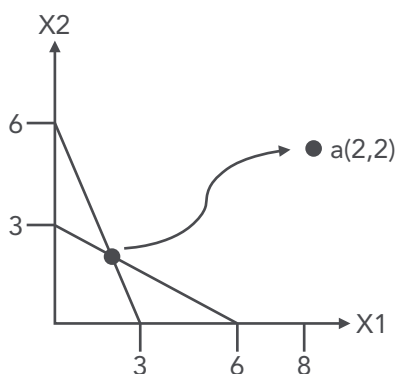
Min. cost = $10X_1 + 40X_2$

S.T.

$2X_1 + 4X_2 = 12$

$4X_1 + 2X_2 = 12$

$X_1, X_2 \geq 0$



Corner point	cost
a (2,2)	100






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Optimal solution is producing 2 unit of X1 and producing 2 units of X2, this decision will realize minimum cost of 100 JD.

2.6 SENSITIVITY ANALYSIS

- It is called post optimality analysis and what if analysis.
- This analysis is investigating the impact of changes in environmental factors on the optimal solution. It is the same as simulation.
- The indicators reflects the changes in environmental factors are;
 - a) The coefficients of objective functions
 - b) The coefficients of constraints
 - c) Constraints limits
- The analyst will investigate the ranges of changes in the previous indicators which maintain the optimal solution the same. This is called range of optimality.

2.6.1 SENSITIVITY ANALYSIS FOR THE COEFFICIENTS OF OBJECTIVE FUNCTION

1. Find the optimal solution

Example

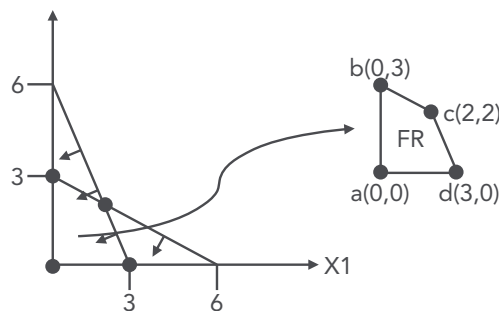
$$\text{Max. profit} = 10X_1 + 40X_2$$

S.T.

$$2X_1 + 4X_2 \leq 12$$

$$4X_1 + 2X_2 \leq 12$$

$$X_1, X_2 \geq 0$$



Corner point	Profit
a (0,0)	0
b (0,3)	120
c (2,2)	100
d (3,0)	30

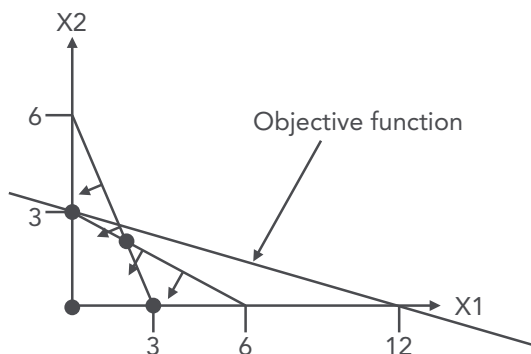
Optimal solution is not producing any unit of X1 and producing 3 units of X2, this decision will realize maximum profit of 120 JD.

2. Draw the objective function on the same graph.

$$120 = 10X_1 + 40X_2$$

Assume $X_1 = 0 \dots \dots \dots X_2 = 3$ (0,3)

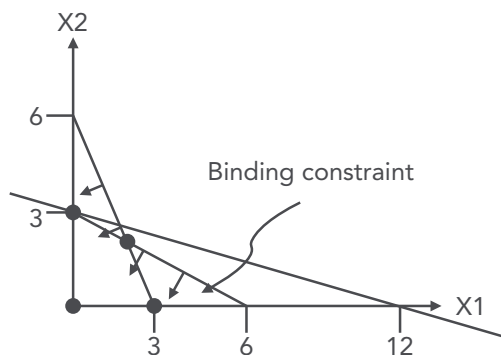
Assume $X_2 = 0 \dots \dots \dots X_1 = 12$ (12,0)



3. Identifying the binding constraint

This constraint has three attributes

- a) Closest constraint for the objective function almost always when it is moved in the same direction of clock wise.
- b) Shares the optimal solution point with the objective function
- c) Represents a boarder for the feasible region



The binding constraint is

$$2X_1 + 4X_2 \leq 12$$

4. Find the slope of objective function and binding constraint

$$\text{Slope} = -C1/C2$$

C1: coefficient of X1 C2: Coefficient of X2.

$$\text{Slope of objective} = -10/40 = -0.25$$

$$\text{Slope of binding constraint} = -2/4 = -0.5$$

5. Compare the slope of objective function with the slope of constraint

$$-0.25 \geq -0.5$$

6. find the range of optimality of X1

Assume the coefficient of X1 in objective function as unknown

$$-C1/40 \geq -2/4 \dots\dots\dots C1 \leq 20$$

Range of C1 is $10 \leq C1 \leq 20$

7. find the range of optimality of X2

Assume the coefficient of X2 in objective function as unknown

$$-10/C2 \geq -2/4 \dots\dots\dots C2 \geq 20$$

Range of C2 is $20 \leq C2 \leq 40$



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2.7 REVIEW QUESTIONS

1. Green Vegetables caned manufacturing corp. produces two kinds of caned pees; check bees and black bees. The production of each kind of caned pees passed through three processes. Pees washing, filling process, packaging process. For each lot size of check pees (1000 cans per lot size) the time required for washing pees is 1 hour, for filling 2 hours and for packaging is 1.5 hours. However, for each lot size of black pees (2000 cans per lot size) the time required for washing pees is 2 hour, for filling 4 hours and for packaging is 3 hours. The total labour time available for washing per month is 800 hours, for filling process is 1200 hours and for packing is 1600 hours. Each can of check pees requires 0.01kg of check pees, and the amount of check pees available next month is 2000 k.g., however, the each can of black pees requires 0.05kg of black pees, and the amount of black pees available next month is 3000 k.g. the manufacturing seeks to identify the number of lot sizes will be produces next month to realize the maximum profit. The profit per lot size of check pees is 2000 J.D. and the profit of black pees is 2400 J.D.
 - a) Could you please formulate the previous problem
 - b) Could you please find the optimal solution

2. Coca Cola corp. produces two kinds of soft drinks; Cola and Vantaa. The production of each kind of drink passed through three processes. Cans stamping, filling process, packaging process. For each lot size of Cola (2500 cans per lot size) the time required for cans stamping is 3 hour, for filling 6 hours and for packaging is 3 hours. However, for each lot size of Vantaa (4000 cans per lot size) the time required for cans stamping is 8 hour, for filling 10 hours and for packaging is 6 hours. The total labour time available for stamping per month is 1200 hours, for filling process is 2400 hours and for packing is 3200 hours. Each can of Cola requires 0.02 Litters of Soda, however, each can of Vantaa requires 0.01 litters of soda, and the amount of Soda available next month is 4000 litter, the number of produced lot sizes of Cola shouldn't be less two times of Vantaa. The manufacturing manager seeks to identify the number of lot sizes will be produces next month to realize the maximum profit. The profit per lot size of Cola is 5000 J.D. and the profit of Vantaa is 3000 J.D.
 - a) Could you please formulate the previous problem
 - b) Could you please find the optimal solution

3. The operations manager of a bank seeks to identify the optimal number of ATMs should be located in Amman and Irbid to serve the maximum number of customers. Each ATM in Amman is expected to serve 20,000 customers a year, and each ATM in Irbid is expected to serve 5,000 customers a year. The number of located ATMs is determined by the budget available and the number of customers expected to be served in each city. Each ATM located in Amman costs about 5000 J.D. and each ATM in Irbid costs 4500 J.D., the budget is up to 120,000 J.D. The number of customers living in the selected location of Amman is 250,000 and in Irbid is 150,000 customers. The number of ATMs in Amman should be two time or equal the number of Irbid.
- Could you please formulate the previous problem
 - Could you please find the optimal solution
4. The operations manager of a hospital seeks to identify the optimal number of beds should be available in two departments; department (A) and department (B) to serve maximum number of patients. Each bed of department (A) serves 60 patients a year and each bed of department (B) serves 50 patients a year. The number beds is determined by the number of nurses and physicians available. Each bed of department (A) requires 2 nurses and each bed of department (B) requires 1 nurse. The number of available nurses is 60. Each bed of department (A) and each bed of department (B) required a physicians. The number of department (A) physicians available is 30 and number of department (B) physicians available is 25. The number of department (A) beds available should be the same or more than the number of department (B) beds.
- Could you please formulate the previous problem
 - Could you please find the optimal solution
5. Solve the following LP problem using the corner point graphical method
- Maximize profit= $14X_1 + 16X_2$
 Subject to;
 $2X_1 + 3X_2 \geq 21$
 $5X_1 + 4X_2 \geq 40$
 $X_2 \geq 5$
 $2X_2 \geq 2X_1 + 2$
 $X_1, X_2 \geq 0$

6. Please find the optimal solution for the following linear programming problem

$$\text{Min. Cost} = 50X_1 + 40X_2$$

S.T.

$$2X_1 + 3X_2 > 21$$

$$5X_1 + 4X_2 > 40$$

$$X_2 > 5$$

$$2X_2 > 2X_1 + 2$$

$$X_1, X_2 > 0$$

7. Solve the following LP problem using the corner point graphical method

$$\text{Maximize profit} = 14X_1 + 16X_2$$

$$\text{Subject to; } 3X_1 + 3X_2 \geq 15$$

$$2X_1 + 4X_2 \geq 28$$

$$X_2 \leq 5$$

$$2X_2 = X_1 + 6$$

$$X_1, X_2 \geq 0$$

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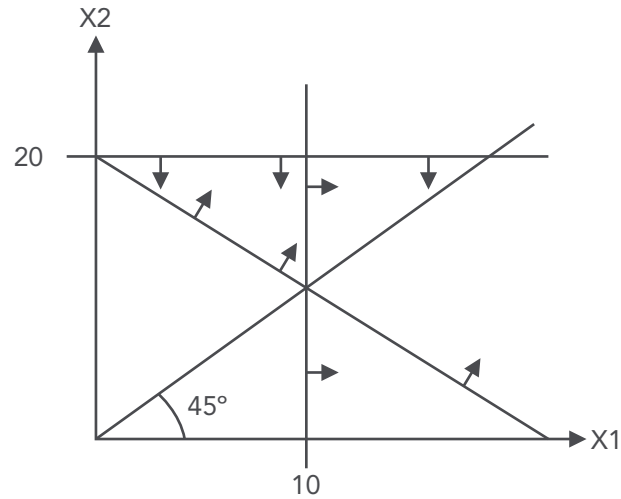
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8. The following is the graphically representation of a linear programming problem. If the objective function of this problem is $\text{Max profit} = 30X_1 + 40X_2$. Please answer the following questions



- Please identify the feasible region on the graphical presentation.
- Please identify the gravities of corner points.
- Please identify the optimal solution.

3 LINEAR PROGRAMMING: INTEGER PROGRAMMING

3.1 LEARNING OUTCOMES

1. Choosing the suitable integer programming technique to solve the required type of integer.
2. Solve the problem by using the chosen integer programming technique.

3.2 DEFINITION

Linear programming integer is transforming the business problem into linear formulas (equations or inequalities) for a problem contains two decision variables in order to reach integer optimal solution.

3.3 ASSUMPTIONS

The same as graphical assumptions.

- After finding the optimal solution the decision outcomes could be fractions, but the fractions are meaning less. This depends on the types of variable scale, whether its discrete or continuous.
- Discrete variable is the same as number of whole unit, as the number of vehicles produced, number of fans, number of TVs...etc.
- Continuous variables is the same as litres of oils, meters of carpets, litres of milk, tons of metal...etc.

3.4 TYPES OF INTEGER PROGRAMMING

- a) Complete: all decision variables should be integer.

For example

$$\text{Max. profit} = 5.79X_1 + 25.52X_2$$

S.T.

$$X_1 \leq 28031 \text{..demand of table fans}$$

$$X_2 \leq 23250 \text{..demand of stand fans}$$

$$0.3 X_1 + 0.4X_2 \leq 11539 \text{..kg of metal}$$

$$0.2 X_1 + 0.46X_2 \leq 560000 \text{..labor hours}$$

$$X_1, X_2 \geq 0, \text{ integer}$$

b) Partial integer: some variables are integers.

For example

$$\text{Max. profit} = 5.79X_1 + 0.50X_2$$

S.T.

$$X_1 \leq 28031 \text{..demand of table fans}$$

$$0.3 X_1 \leq 11539 \text{..kg of metal}$$

$$0.2 X_1 + 0.06X_2 \leq 560000 \text{..labor hours}$$

$$X_1, X_2 \geq 0, X_1: \text{integer}$$

In the previous example X_1 is the number of produced fans but X_2 is meters of cables.
So X_1 integer and X_2 could be non integer.

c) Zero one integer: the whole decision could be chosen (1) or ignored (0).

0: don't 1: Do

It is widely used for investment decision

$$\text{Max. NPV} = 25000X_1 + 35000X_2$$

S.T.

$$12000X_1 + 10000X_2 \leq 15000 \text{...First year investment}$$

$$10000X_1 + 8000X_2 \leq 16000 \text{...Second year investment}$$

$$X_1, X_2 = 0, 1$$



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3.5 TECHNIQUES TO SOLVE THE INTEGER PROBLEMS

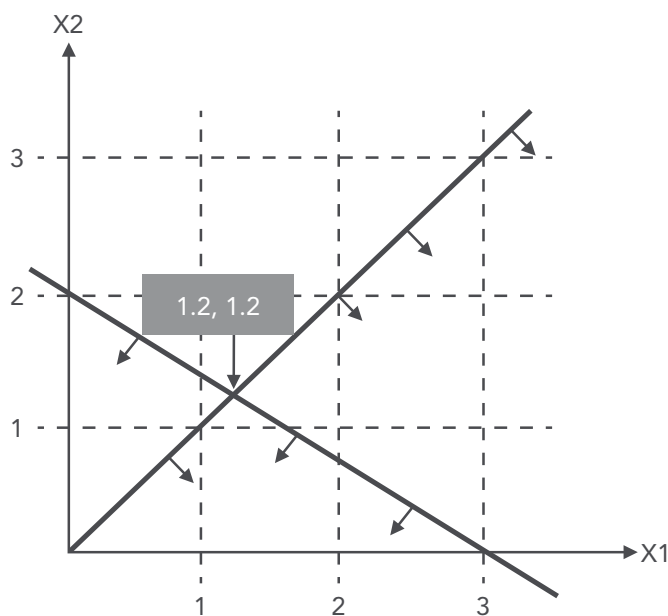
Two widely used techniques, which are; graphical and branches and bounds.

Matrix of techniques and types of integer

Types of integer	Graphical	Branches and bounds
Complete	Yes Used when the scale of variables is single numbers (1-9)	Yes used when the scale of variable is 10 th , 100, 1000... etc
Partial (mixed)	-----	Yes
0,1	Yes	Yes

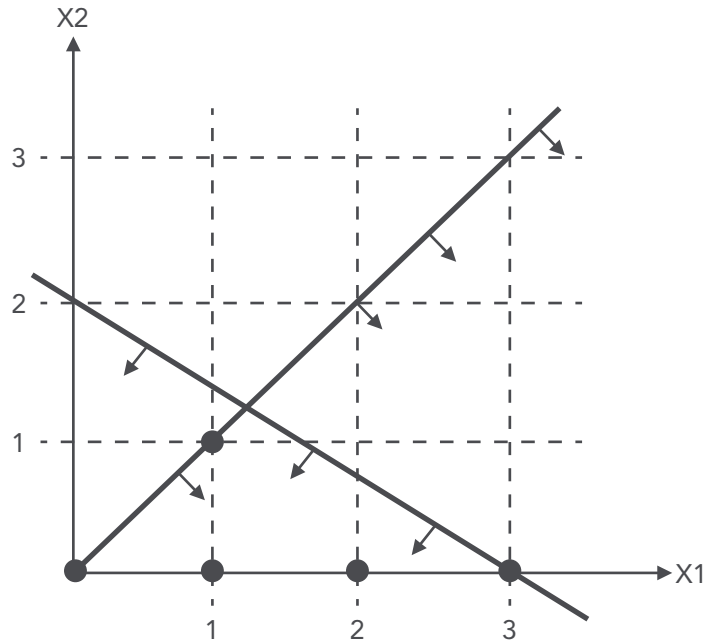
3.5.1 GRAPHICAL SOLUTION

1. Find the optimal solution graphically



The optimal solution is $X_1=1.2$ and $X_2=1.2$ and profit =13.2, the objective function is Max. Profit= X_1+10X_2 .

2. Identifying the integer points within the feasible region



Integer points within feasible region are;
 (0,0) (1,0) (2,0) (3,0) (1,1)

3. Identifying the optimal integer point

Integer point	profit
(0,0)	0
(1,0)	1
(2,0)	2
(3,0)	3
(1,1)	11.....optimal point

The optimal integer solution is producing 1 unit of X1 and producing 1 unit of X2 and realized profit is 11.

3.5.2 BRANCHES AND BOUNDS

1. Find the optimal solution graphically

Max. Profit= $X_1 + 10X_2$

S.T.

$2X_1 + 3X_2 \leq 6$

$X_1 \geq X_2$

$X_1, X_2 > 0, X_2: \text{integer}$

The optimal solution is $X_1 = 1.2$, $X_2 = 1.2$ and total profit = 13.2, the result will be represented on a bound.

A

$X_1 = 1.2$

$X_2 = 1.2$

$P = 13.2$

2. Start with any integer variable if the problem is complete, but if its partial and only one variable you should start with this variable.

In this problem the integer is only X_2 so the problem is partial.

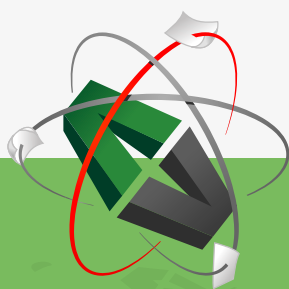
Non integer value of X_2 is 1.2.

What is the closest integer value more than 1.2 and less than 1.2?

The values are 1 lower value and 2 upper value. It is not acceptable to have a value between 1 and 2. The value should be more than or equal 2 or less than or equal 1.

Accordingly the previous problem will be divided into two problem as following

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Problem (B)

Max. Profit= X_1+10X_2

S.T.

$2X_1+3X_2 \leq 6$

$X_1 \geq X_2$

$X_1 \leq 1$

 $X_1, X_2 > 0$, X_2 : integer**Problem (C)**

Max. Profit= X_1+10X_2

S.T.

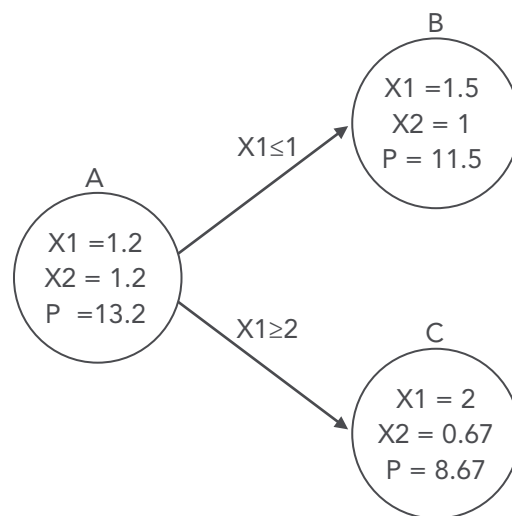
$2X_1+3X_2 \leq 6$

$X_1 \geq X_2$

$X_1 \geq 2$

 $X_1, X_2 > 0$, X_2 : integer

Each sub-problem will be solved graphically and the optimal solution of each problem will be presented on a bound as following



Continue the same procedure till all non-integers be integers.

3. Identifying the integer optimal solution

In the previous case the integer optimal solution is the outcome of problem (B), $X_1=1.5$, $X_2=1$ and $P=11.5$

3.6 REVIEW QUESTIONS

1. The following is a complete integer linear programming problem. Please solve it by using the graphical method. (The optimal non integer solution is $X_1 = 3.375, X_2 = 2.25$) (7 Mark)

Max. Profit = $3X_1 + 2X_2$

s.t.

$$2X_1 + 5X_2 \leq 18$$

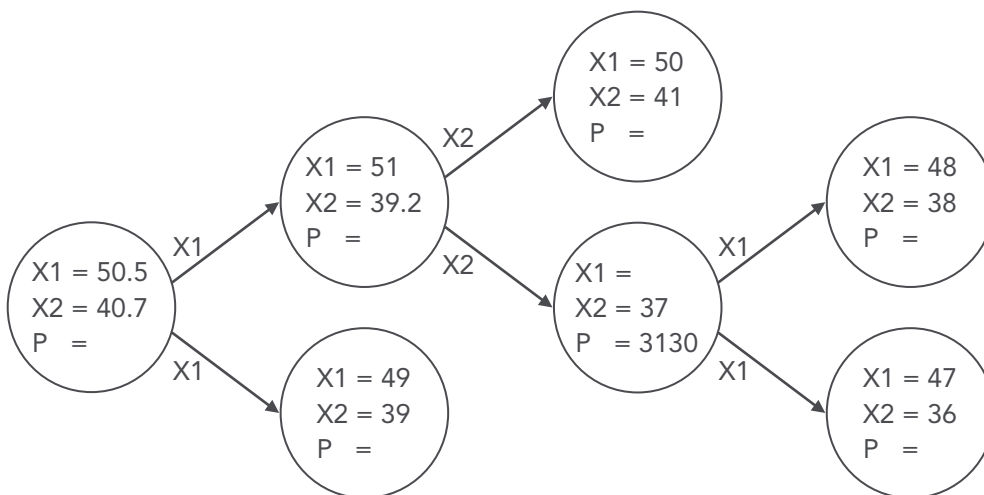
$$4X_1 + 2X_2 \leq 18$$

$$X_1, X_2 \geq 0, \text{ integers}$$

2. If the objective function of a complete integer programming problem is

Max Profit = $50X_1 + 20X_2$

The bonds and branches technique was used to find the optimal the solution, please complete the following branches and bonds diagram and find the optimal solution.



4 LINEAR PROGRAMMING: SIMPLEX METHOD

4.1 LEARNING OUTCOMES

1. Developing the initial simplex table
2. Improving the simplex plans till reach the optimal simplex plan
3. Make the decision according to final simplex table
4. Conduct the sensitivity analysis for the outcomes of final simplex table

4.2 DEFINITION

Quantitative approach of decision making which transforms business problems into linear formulas (equations or in equalities), for a problem contains two or more decision variables in order to reach the optimal solution.

4.3 ASSUMPTIONS

The same as graphical assumptions except one assumption; the number of decision variables is two or more.



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4.4 HOW TO MAKE THE REQUIRED SETUP FOR THE PROBLEM

it's a phase done before start the solving of problem. It is called standardisation phase; writing the problem as a standard format

- a) Transform all inequalities into equations
 - a-1 if the direction of inequality is less than or equal then add a slack variable (+Si).
 - a-2 if the direction of inequality is more than or equal then deducted surplus (-Si) and add artificial variable (+Ai).
- b) Rewrite the equations by adding an artificial variable (+Ai).
- c) Rewrite the objective function; all variables in the problem should appear in the objective function, so the variables that imposed in the inequalities and equations should occur in the objective function.
 - c-1 The slack and surpluses should be added to objective function as (+0Si).
 - c-2 The artificial variables are added or deducted as follows

If the problem is Max., the artificial will be deducted as (-MAi)

If the problem is Min., the artificial will be added as (+MAi).

Example:

Could you please make the required setup for the following problem

$$\text{Max.} = 100X_1 + 200X_2 + 150X_3$$

S.T

$$6X_1 + 2X_2 + 8X_3 \leq 100$$

$$4X_1 + 1X_2 + 6X_3 \geq 120$$

$$X_1 = 2X_2$$

$$X_1, X_2, X_3 \geq 0$$

$$\text{Max.} = 100X_1 + 200X_2 + 150X_3 + 0S_1 + 0S_2 - MA_1 - MA_2$$

S.T

$$6X_1 + 2X_2 + 8X_3 + S_1 = 100$$

$$4X_1 + 1X_2 + 6X_3 - S_2 + A_1 = 120$$

$$X_1 - 2X_2 + A_2 = 0$$

$$X_1, X_2, X_3, S_1, S_2 \geq 0$$

If the problem is Min. Instead of Max. Could you please make the required setup for the problem.

$$\text{Min.} = 100X_1 + 200X_2 + 150X_3 + 0S_1 + 0S_2 + MA_1 + MA_2$$

S.T

$$\begin{aligned}
6X_1+2X_2+8X_3+S_1 &= 100 \\
4X_1+1X_2+6X_3-S_2+A_1 &= 120 \\
X_1-2X_2+A_2 &= 0 \\
X_1, X_2, X_3, S_1, S_2 &\geq 0
\end{aligned}$$

4.5 DEVELOPING THE INITIAL SIMPLEX TABLE

- d) Make the required setup for the problem (phase 1).
- e) Draw a table contains row equal the number of constraints, plus two rows at the top and the same at the bottom.
- f) The table contains columns equal the number of variables in the objective function after setup, plus two columns at beginning and one at the end.
- g) Then, in the first two rows at the top write the decision variables of the objective function and their coefficients.
- h) Under each variable write it's coefficient as appears in each constraint. The coefficients are located in the cells of each constraint row.
- i) Assume $X_1, X_2, X_3=0$, start from the origin point. The metaphor is taking a picture for the factory during shutdown.
- j) According to the previous assumption, the decision is maintaining the resources without usage. So in the second column from left will contain surplus, slack or artificial. This column is called solution mix. column.
- k) In the first column which corresponds the solution mix column the coefficient of solution mix column variables of the objective function are identified in the first column.
- l) Then compute the profit realised by the solution mix (Z_j).
- m) Compute the net profit realised by problem variables in comparison with solution mix (C_j-Z_j).

Example:

$$\text{Max. Profit} = 100X_1 + 200X_2 + 150X_3$$

S.T

$$\begin{aligned}
6X_1+2X_2+8X_3 &\leq 100 \dots \text{Labour hours of cars painting} \\
4X_1+1X_2+6X_3 &\leq 120 \dots \text{labour hours of cars body shop} \\
6X_1+2X_2+4X_3 &\leq 200 \dots \text{labour hours of cars assembly} \\
X_1, X_2, X_3 &\geq 0
\end{aligned}$$

Problem setup

$$\text{Max. Profit} = 100X_1 + 200X_2 + 150X_3 + 0S_1 + 0S_2 + 0S_3$$

S.T

$$6X_1 + 2X_2 + 8X_3 + S_1 = 100 \dots \text{Labour hours of cars painting}$$

$$4X_1 + 1X_2 + 6X_3 + S_2 = 120 \dots \text{labour hours of cars body shop}$$

$$6X_1 + 2X_2 + 4X_3 + S_3 = 200 \dots \text{labour hours of cars assembly}$$

$$X_1, X_2, X_3, S_1, S_2, S_3 \geq 0$$

Cj	Solution mix	100	200	150	0	0	0	Quantity
		X1	X2	X3	S1	S2	S3	
0	S1	6	2	8	1	0	0	100
0	S2	4	1	6	0	1	0	120
0	S3	6	2	4	0	0	1	200
Zj		0	0	0	0	0	0	0
Cj-Zj		100	200	150	0	0	0	

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4.6 IMPROVING THE SIMPLEX TABLE

4.6.1 GENERAL METHOD

Two in transition questions should be asked;

Q1 – Which variable will enter into the solution mix? **Or which product will be produced?**

Look at $C_j - Z_j$ column, this column represents the profit realised by each unit of the product in comparison with the solution mix. Logically the product which realises max. profit per unit will be produced. In the previous case X2 will be produced since it realises max. profit which is 200. The column of this variable is called pivot column. Since the numbers of this column are pivotal and will be used to compute the ratio values later. This column is shaded.

Q2 – Which variable will leave the solution mix? **Or which resource will be fully utilized?**

In order to identify this variable or resource the ratio will be computed. The ratio is dividing the quantity over the corresponding number of pivot column. The ratio means if the total resources will be directed toward producing the best product, what is the total production size?

In previous example the ratio are;

Cj	Solution mix	200	Quantity	Ratio
		X2		
0	S1	2	100	100/2 = 50 cars of X2
0	S2	1	120	120/1= 120 cars of X2
0	S3	2	200	200/2= 100 cars of X2
Zj		0	0	
Cj-Zj		200		

The minimum ratio will be identified. This is logical since the production of cars should pass the three processes, body shop, painting and assembly. So if 120 boys are produced, the painting process capacity is only 50 cars, so the difference which is 70 bodies will be stored in the warehouses, if 70 bodies will be stored daily, the inventory will be built up till reach huge volume. The inventory holding cost will increase and the bodies will not be transformed to cars. Accordingly, the max. Capacity of the Production process is the bottle neck value. This situation could be managed by using theory of constraints (TOC). The shaded row corresponds the minimum ratio is called pivot row, since it's numbers will be the pivotal in developing the next table.

Q3 – What is the size of production of X1,X2, X3 of the next production plan?

All resources will be directed toward producing X2 only. X1 and X3 will not be produced. (Note: in each improved table only one product will enter into the production mix. So the size of production is X1=0, X3=0, X2=50 (the same as minimum ratio).

Q4 – What is the total profit in the next table (production plan)?

Size of production of X2 × profit per unit = 50 × 200 = 10,000 J.D.

Q5 – How to modify the pivot row?

Divide all numbers in the pivot row by the pivot number (the pivot number which is the number of intersection cell between pivot row and column).

Cj	Solution mix	100	200	150	0	0	0	Quantity
		X1	X2	X3	S1	S2	S3	
0	S1	6/2=3	2/2=1	8/2=4	1/2	0/2=0	0/2=0	100/2=50

Q6 – How to modify other rows rather than pivot row?

The following formula will be used;

Net resources = total available resources – total used resources.

Modified number = number in the old table – (number above or below pivot number × corresponding number of modified pivot row).

S2 raw

$$4 - (1 * 3) = 1$$

$$1 - (1 * 1) = 0$$

$$6 - (1 * 4) = 2$$

$$0 - (1 * \frac{1}{2}) = -\frac{1}{2}$$

$$1 - (1 * 0) = 1$$

$$0 - (1 * 0) = 0$$

$$120 - (1 * 50) = 70$$

S3 raw

$$6 - (2 * 3) = 0$$

$$2 - (2 * 1) = 0$$

$$4 - (2 * 4) = -4$$

$$0 - (2 * \frac{1}{2}) = -1$$

$$0 - (2 * 0) = 0$$

$$1 - (2 * 0) = 1$$

$$200 - (2 * 50) = 100$$

The following is the modified table (next table).

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Cj	Solution mix	100	200	150	0	0	0	Quantity
		X1	X2	X3	S1	S2	S3	
200	X2	3	1	4	1/2	0	0	50
0	S2	1	0	2	-1/2	1	0	70
0	S3	0	0	-4	-1	0	1	100
Zj		600	200	800	100	0	0	10,000
Cj-Zj		-500	0	-650	-100	0	0	

4.6.2 BIG (M) METHOD

This technique is used when the constraints are inequalities, it's direction is more than or equal, or the constraint is equation. In these cases the artificial variable will be added for the objective function with M coefficient, so the variables will enter the solution mix when it's M value is the biggest.

Example 1: Max. Problem

$$\text{Max.} = 100X_1 + 200X_2 + 150X_3$$

S.T

$$6X_1 + 2X_2 + 8X_3 \leq 100$$

$$4X_1 + 1X_2 + 6X_3 \geq 120$$

$$X_1 = 2X_2$$

$$X_1, X_2, X_3 \geq 0$$

Could you please developing the initial simplex table?

1. Make the required setup for the problem

$$\text{Max.} = 100X_1 + 200X_2 + 150X_3 + 0S_1 + 0S_2 - MA_1 - MA_2$$

S.T

$$6X_1 + 2X_2 + 8X_3 + S_1 = 100$$

$$4X_1 + 1X_2 + 6X_3 - S_2 + A_1 = 120$$

$$X_1 - 2X_2 + A_2 = 0$$

$$X_1, X_2, X_3, S_1, S_2 \geq 0$$

Cj	Solution mix	100	200	150	0	0	-M	-M	Quantity
		X1	X2	X3	S1	S2	A1	A2	
0	S1	6	2	8	1	0	0	0	100
-M	A1	4	1	6	0	-1	1	0	120
-M	A2	1	-2	0	0	0	0	1	0
Zj		-5M	+M	-6M	0	+M	-M	-M	-120M
Cj-Zj		100+5M	200-M	150+6M	0	-M	0	0	

Q1 – Which variable will enter into the solution mix? Or which product will be produced?

Look at Cj-Zj values, identifying the positive values of (M), this means the profit will be increased by M, the biggest value is the maximum increase in profit.

X1: 100+5M

X3: 150+6M...biggest value of (M). So X3 will be produced. Then compute the ratio as discussed earlier.

Example 2: Min. Problem

$$\text{Min.} = 100X_1 + 200X_2 + 150X_3$$

S.T

$$6X_1 + 2X_2 + 8X_3 \leq 100$$

$$4X_1 + 1X_2 + 6X_3 \geq 120$$

$$X_1 = 2X_2$$

$$X_1, X_2, X_3 \geq 0$$

Could you please developing the initial simplex table?

1. Make the required setup for the problem

$$\text{Min.} = 100X_1 + 200X_2 + 150X_3 + 0S_1 + 0S_2 + MA_1 + MA_2$$

S.T

$$6X_1 + 2X_2 + 8X_3 + S_1 = 100$$

$$4X_1 + 1X_2 + 6X_3 - S_2 + A_1 = 120$$

$$X_1 - 2X_2 + A_2 = 0$$

$$X_1, X_2, X_3, S_1, S_2 \geq 0$$

Cj	Solution mix	100	200	150	0	0	+M	+M	Quantity
		X1	X2	X3	S1	S2	A1	A2	
0	S1	6	2	8	1	0	0	0	100
+M	A1	4	1	6	0	-1	1	0	120
+M	A2	1	-2	0	0	0	0	1	0
Zj		+5M	-M	+6M	0	-M	+M	+M	+120M
Cj-Zj		100-5M	200+M	150-6M	0	+M	0	0	

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Q1- Which variable will enter into the solution mix? Or which product will be produced? Look at $C_j - Z_j$ values, identifying the negative values of (M), this means the cost will be decreased by M, the biggest value is the max. Reduction in cost.

$X_1: 100 - 5M$

$X_3: 150 - 6M$...biggest value of (M). So X_3 will be produced. Then compute the ratio as discussed earlier.

Note: the values of ratio should be considered

- The ratio should be considered is only the positive. Negative, zero and infinity is not considered.

Example:

X3	Q	Ratio
2	100	$100/2=50$...consider
0	200	$200/0=$ infinity...ignore
-2	120	$120/-2=-60$...ignore
2	0	$0/2=0$...ignore

- If two values of $C_j - Z_j$ has the same biggest value of (M), the analyst will choose the variable with maximum attached number with (M) if the problem is Max. or minimum attached number with (M) if the problem is Min.

Example (1) $C_j - Z_j (X_1) = 150 + 6M$

$C_j - Z_j (X_2) = 200 + 6M$chosen variable

Example (2) $C_j - Z_j (X_1) = 150 - 6M$chosen variable

$C_j - Z_j (X_2) = 200 - 6M$

4.7 SENSITIVITY ANALYSIS

4.7.1 RANGES OF RIGHT HAND SIDE VALUE: CASE OF SURPLUS

Cj	Solution mix	100	200	150	0	0	0	Quantity
		X1	X2	X3	S1	S2	S3	
200	X2	3	1	4	1/2	0	0	50
0	S2	1	0	2	-1/2	1	0	70
Zj		600	200	800	100	0	0	10,000
Cj-Zj		-500	0	-650	-100	0	0	

Q1 – Is the previous table the optimal one? Justify.

Yes, look at Cj-Zj, the problem is Max. Since no positive value in this row so the table is optimal.

Q2 – What is the optimal solution?

Producing 50 units of X2 and not produce any unit of X1 and X3. The realised profit is 10,000 J.D.

Q3 – What are the resources used to realize the optimal solution?

To identify these values; identifying Si column which contains positive and negative values.

Solution mix.	0	Quantity
	S1	
X2...	1/2	...50
S2...	-1/2	...70
Zj...	100	
Cj-Zj...	-100	

According to this column

- Each $\frac{1}{2}$ car of X2 requires $\frac{1}{2}$ hour of S2 to be produces.
- Each hour of S1 produces $\frac{1}{2}$ car of X2.

X2	S2
$\frac{1}{2}$	$\frac{1}{2}$
50 cars	??? (A)

The used hours of S2 = 50 hours

S1	X2
1	$\frac{1}{2}$
??? (B)	50

The used hours of S1 = 100 hours.

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Q4 – Is there any surplus resources? Could you please identify the resource and the size of surplus?

S2 is the surplus and the size is 70 hours.

Q5 – If the surplus resources will be used. What is the impact of this action on the size of optimal solution?

Return back to the relationship between S2 and X2. and find the impact.

X2	S2
1/2	1/2
???(C)	70

After using the surplus hours of S2 the production size will be increase by 70 cars, so the size of production will be $50+70 = 120$ cars of X2.

Q6 – What is the impact of this action on the fully utilised resource?

The fully utilised resource is S1. In order to find the impact return back to the relationship between S1 and X2

S1	X2
1	1/2
???(D)	+70

The extra hours required are +140 hours.

Q7 – What are the ranges of optimality for the right hand size values?

	Lower bound	Upper bound
X2	50	$50+70=120$
S1	100	$100+140= 240$
S2	50	$50+70=120$

Q8 – What is the return of each extra hour of fully utilised resource?

Identifying the shadow price; which is the negative values of $C_j - Z_j$ under S_1 . The value in -100, this value is the opportunity loss of not hiring extra hour of S_1 .

Q9 – Is it feasible to hire extra hours of S_1 ; if the following recruiting alternatives are available?

Full time employee wage per hour is 120 J.D.

Part time employee wage per hour is 100 J.D.

Overtime employee wage per hour is 80 J.D.

The alternative is feasible if the cost of alternative is less than the shadow price, so the feasible alternative is overtime employee.

Q10 – What is the total return of S_1 if 10 extra hours will be hired?

Net return per hour of overtime is= $100 - 80 = 20$ J.D./hour

Total return= $10 * 20 = 200$ J.D.

4.7.2 RANGES OF RIGHT HAND SIDE VALUE: CASE OF NO SURPLUS

Max. Profit = $7X_1 + 8X_2 + 10X_3$

S.T.

$2X_1 + 3X_2 + 2X_3 \leq 1000$

$1X_1 + 1X_2 + 2X_3 \leq 800$

$X_1, X_2, X_3 \geq 0$

Cj	Solution mix	7	8	10	0	0	Quantity
		X1	X2	X3	S1	S2	
7	X1	1	2	0	1	-1	200
10	X3	0	-1/2	1	-1/2	1	300
Zj		7	9	10	2	3	4400
Cj-Zj		0	-1	0	-2	-3	

Q1 – Is the previous table the optimal one? Justify.

Yes, since the $C_j - Z_j$ not contains any positive value.

Q2 – What is the optimal solution?

Producing 200 units of X_1 , 300 units of X_3 , and not producing any unit of X_2 . The realised profit is 4400 J.D.

Q3 – What are the resources used to realize the optimal solution?

Just substitute the values of X1 and X3 in the first and second constraint.

The values are 1000 hours of S1 and 800 hours of S2. So the resources are fully utilised.

Q4 – If the manager decided to produce X1 and stop producing X3. What is the impact of this action on the size of production of X1.

X1 replaced S1 row. Identifying Si column at which X1 is positive and X3 is negative. This is column S1.

Solution mix	0	Q
	S1	
X1...	1	...200
X3...	-1/2	...300
	2	
	-2	

Each half fan reduced of X3 increase the fan of X1 by 1.

So the size of production will be increased by 600 fans.

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Q5 – What is the impact of this action on S1?

Return back to the relationship between S1 and X1.

Each hour of S1 produces 1 fan of X1.

So 600 extra fans require 600 hours of S1.

Q6 – If the manager decided to produce X3 and stop producing X1. What is the impact of this action on the size of production of X3.

X3 replaced S2 raw. Identifying Si column at which X3 is positive and X1 is negative. This is column S2.

Solution mix	0	Q
	S2	
X1...	-1	...200
X3...	1	...300
	3	
	-3	

Each fan reduced of X1 increase the fan of X3 by 1.

So the size of production will be increased by 200 fans.

Q7 – What is the impact of this action on S2?

Return back to the relationship between S2 and X3.

Each hour of S2 produces 1 fan of X1.

So 200 extra fans require 200 hours of S2.

Q8 – What are the ranges of optimality for the right hand size values?

	Lower bound	Upper bound
X1	200	200+600=800
S1	1000	1000+600= 1600
X3	300	300+200=500
S2	800	800+200=1000

Q9 – What is the return of each extra hour of S1 and S2?

The shadow price of S1 is 2 J.D. and the shadow price of S2 is 3 J.D.

Q10 – Is it feasible to hire part time employee if the wage of each hour is 1 J.D.

For S1 and S3 is feasible, since the cost is less than the shadow price.

Net return per hour of S1= 2-1=1 J.D.

Net return per hour of S2= 3-1= 2 J.D.

4.7.3 THE RANGE OF OPTIMALITY FOR THE COEFFICIENTS OF OBJECTIVE FUNCTION

The following example will be used to explain the procedure

Cj	Solution mix	7	8	10	0	0	Quantity
		X1	X2	X3	S1	S2	
7	X1	1	2	0	1	-1	200
10	X3	0	-1/2	1	-1/2	1	300
Zj		7	9	10	2	3	4400
Cj-Zj		0	-1	0	-2	-3	

1. Reach the optimal solution; previous table is optimal one since $C_j - Z_j$ not contains any positive value.
2. Identifying the basic decision variables; the basic variables are the solution mix. variables, the concern is about the basic decision variables, so the surplus variable (S_i) will be out of concern, since it's not a decision variable and its coefficient is zero, so no range of optimality for this variable. In previous example the basic decision variables are X1 and X3.
3. Add symbol of change like (D) for the coefficients of one basic variable and find the value of Zj and $C_j - Z_j$.

Cj	Solution mix	7+D	8	10	0	0	Quantity
		X1	X2	X3	S1	S2	
7+D	X1	1	2	0	1	-1	200
10	X3	0	-1/2	1	-1/2	1	300
Zj		7+D	9+2D	10	2+D	3-D	
Cj-Zj		0	-1-2D	0	-2-D	-3+D	

4. Find the values of D; this will be found through the $C_j - Z_j$ for non basic variables.
 In previous case the non basic variables are X2, S1 and S2.
 $C_j - Z_j$ for non basic is ≤ 0

Value of D range of C1

$C_j - Z_j (X2) \leq 0 \dots -1 - 2D \leq 0 \dots D \geq -1/2 \dots 6.5 \leq C1$

$C_j - Z_j (S1) \leq 0 \dots -2 - D \leq 0 \dots D \geq -2 \dots 5 \leq C1 \dots$ wider low range

$C_j - Z_j (S2) \leq 0 \dots -3 + D \leq 0 \dots D \leq 3 \dots C1 \leq 10 \dots$ wider upper range

So in general the range of C1 is; $5 \leq C1 \leq 10$

Cj	Solution mix	7	8	10+D	0	0	Quantity
		X1	X2	X3	S1	S2	
7	X1	1	2	0	1	-1	200
10+D	X3	0	-1/2	1	-1/2	1	300
Zj		7	9-D/2	10+D	2-D/2	3+D	
Cj-Zj		0	-1+D/2	0	-2+D/2	-3-D	

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$$\begin{aligned}
 -1+D/2 \leq 0 \dots D \leq 2 \dots C3 \leq 12 \\
 -2+D/2 \leq 0 \dots D < 4 \dots C3 \leq 14 \\
 -3-D \leq 0 \dots D \geq -3 \dots C3 \geq 7
 \end{aligned}$$

So the range of C2 is; $7 \leq C3 \leq 14$

4.8 REVIEW QUESTIONS

- The following is the simplex table of a maximize profit problem. The variables X1, X2, X3 are the washing machines produced at LG factory. The production of washing machine passes two processes the framing of bodies and the assembly of parts. The assembly working hours is presented by S1 and the framing working hours are presented by S2. Please answer the following questions.

Cj	S.M.	200	100	150	0	0	Q
		X1	X2	X3	S1	S2	
200	X1	1	2	1	¼	0	100
0	S2	0	1/2	1	-1	1	50
Zj							
Cj-Zj							

- Is the previous table the optimal one? Justify your answer.
- How many framing hours used to realize the optimal solution? (shows the calculations).
- How many extra hours of assembly required if the production manager decided to fully utilize the framing working hours?
- If the production manager decided to hire an assembly employee to increase the assembly hours by 10 hours a day. What is the net return of this action if the wage of employee per hour is 10 J.D.? (Shows the calculations)
- What is the range of optimality for the coefficients of objective function?

- The following is a simplex problem, please answer the following questions

Min. Cost= $20 X1+30X2+40X3$

S.T.

$$\begin{aligned}
 2X1+3X2+4X3 &\geq 120 \dots \text{painting hours} \\
 X1+2X2+5X3 &\leq 100 \dots \text{engine building hours} \\
 X1, X2, X3 &\geq 0
 \end{aligned}$$

- a) Make the required setup for this problem?
- b) Please develop the initial simplex table

3. If the following is a row of a simplex table. The objective function of this problem is $\text{Max profit} = 30X_1 + 40X_2 + 60X_3$. The constraints of this problem are two, the constraints are inequalities with signs \leq , please answer the following questions.

Zj	28	40	80	2	3	2400
----	----	----	----	---	---	------

- a) Is the previous table the optimal one? Justify your answer
- b) What is the variable will enter into the solution? Justify your answer
- c) If the ratios of variable will enter the solution are 60 and 120. Answer the followings
 - c-1 The amount will produced from this decision variable is.....
 - c-2 The profit will be increased by.....

5 TRANSPORTATION PROBLEM

5.1 LEARNING OUTCOMES

1. Formulating the transportation problem as a linear programming problem.
2. Developing the initial transportation table by using North West Corner method and Vogel's Approximation method.
3. Improving the transportation table by using Stepping Stone method and Modified Distribution method.
4. Solving special cases of transportation problem.

5.2 DEFINITIONS AND INTRODUCTION

- Transportation: it is the movement of product, primary inputs, resources, and end of product life cycle from origin to destination.
- Transportation decision
 - a) Selecting routes of transportation: it could be single, or tree or network.
 - b) Mode of transportation: rail, truck, flight, shipment and pipeline.
 - c) Direction: forward or backward or in both directions.

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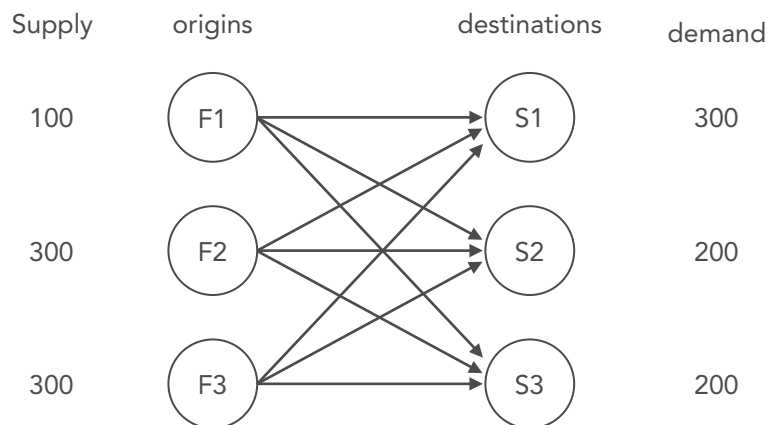
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- Transportation success indicators: time, cost, quality and flexibility.
- The transportation problem is a linear programming problem concerns about selecting the suitable routes to make forward transfer from original to destination in order to reduce total transportation costs.

5.3 ASSUMPTIONS

- The number of decision variables is two or more; the decision variable is identifying the amount will be transferred from each origin to destination.
 X_{11} : amount will be transferred from original 1 to destination 1
 X_{12} : amount will be transferred from original 1 to destination 2
- One objective will be realised which is the amount will be transferred from origins to destinations to minimize total transportation cost.
- The transportation cost of the transferred unit from each origin to each destination should be known.
 Min. Transportation cost= $5X_{11}+8X_{21}+9X_{31}+4X_{12}+4X_{22}+7X_{32}+3X_{13}+3X_{23}+5X_{33}$
- The Constraints are the supply of origins and demand of destinations.
 $X_{11}+X_{12}+X_{13} \leq 100$...supply of origin 1
 $X_{21}+X_{22}+X_{23} \leq 300$...supply of origin 2
 $X_{31}+ X_{32}+X_{33} \leq 300$...supply of origin 3
 $X_{11}+ X_{21}+X_{31} = 300$...Demand of destination 1
 $X_{12}+X_{22}+ X_{32} = 200$...Demand of destination 2
 $X_{13}+X_{23}+ X_{33} = 200$...Demand of destination 3
- Total supply should equal total demand, if they are not equal an artificial variable will be added.
- The values of decision variables should be positive or zero. Since it is impossible to transfer negative values.

5.4 HOW TO DEVELOP THE INITIAL TRANSPORTATION TABLE



The following table shows the transportation cost per unit from origin to destination

	S1	S2	S3
F1	5	4	3
F2	8	4	3
F3	9	7	5

5.4.1 NORTH WEST CORNER METHOD

- a) Start from the cell located on the north west corner.
- b) Allocate as possible as you can via this cell.
- c) Move towards south on the same column, and allocate as possible as you can via the cells.
- d) Move towards the next column and starts from North towards south and allocate as possible as you can via the cells. till allocate all units.

The following is the initial transportation table by using North West Corner method.

Total transportation cost = $100 \times 5 + 200 \times 8 + 100 \times 4 + 100 \times 7 + 200 \times 5 = 4200$ J.D.

	S1	S2	S3	Supply
F1	5 100	4	3	100
F2	8 200	4 100	3	300
F3	9	7 100	5 200	300
Demand	300	200	200	700

5.4.2 VOGEL'S APPROXIMATION METHOD

- This technique is better than North West corner since it depends on opportunity cost in allocating units from origins to destinations.
- It is called Vogel since it was developed by Vogel.
- 80 percent of cases solved by Vogel reach the optimal solution from the initial table.

- Procedure:
 - a) Identifying the two lowest costs corresponds each origin and destination.
 - b) Compute the difference between these two costs; the result is the opportunity cost of not choosing the best alternative.
 - c) Identifying the max. opportunity cost.
 - d) Identifying the best route corresponds the max. opportunity cost. The best which realises the lowest transportation cost.
 - e) Allocate as possible you can via this cell.
 - f) Repeat the procedure till allocate all units.

The following is the initial table developed by using Vogel's approximation method

Total transportation cost = 3900 J.D.



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Opportunity Cost	1	3	2	Supply	Opportunity Cost
	1	0	2		
	3	0	0		
	S1	S2	S3		
F1	5	4	3	100	1
	100	X	X	0	
F2	8	4	3	300	1 1 5
	X	200	100	100	
F3	9	7	5	300	224
	200	X	100	200	
Demand	300	200	200	700	
	200	0	100		
	0		0	700	

5.5 HOW TO IMPROVE THE SOLUTION

- Is it possible to reduce the transportation cost as a result of transferring via empty cells (initially ignored alternatives) in comparison with occupied cells (initially chosen alternatives).
- In order to make such judgement the analyst should has a index. This index is called an improvement index.
- The improvement index is a number which represents the changes in transportation cost per unit as result of transferring via empty cell in comparison with occupied cells.
- The changes could be: positive, negative or zero. Positive index means increase in cost, negative means reduction in transportation cost, but zero it means no change in cost.
- The analyst concerns about negative indices, and concern about the best negative, which realises the maximum reduction in transportation cost per unit.
- The index is represented by a symbol (I_{ij}).
- After identifying the best empty cell, the analyst should identify the amount will be transferred via the best cell.

- So any technique will be adopted should help in;
 - a) Computing the improvement indices of empty cells, and identifying best empty cell.
 - b) Identifying the amount will be transferred via the best empty cell.
- The techniques could be adopted are;
 - a) Stepping Stone method.
 - b) Modified Distribution method (MODI).

Exercise: the followings are the improvement indices of empty cells. Could you please answer the following questions.

$$I_{12}=+3 \quad I_{13}=0 \quad I_{23}=-6 \quad I_{31}=-4$$

- a) Could you please interpret these indices?
- b) Which cell is the best? Justify.
- c) If 200 units will be transferred via the best cell, what is the total reduction in transportation cost?
- d) If the total transportation cost is 4200 JD., what will the total transportation cost be after transferring via the best cell?

5.5.1 FIRST IMPROVEMENT METHOD: STEPPING STONE

5.5.1.1 How to compute the improvement indices

- a) Check the number of occupied cells. The number of occupied cells = number of origin + number of destinations – 1. If the number is less than this number the analyst will be unable to compute improvement indices, and this case is called degeneracy, it will be discussed later.
- b) Start from any empty cell. Transfer one unit via the empty cell (+1 unit via empty cell). Transferring one unit since the improvement indices will be computed for one transferred unit.
- c) Add and deduct one unit from the surrounding occupied cells vertically and horizontally to maintain the balance in supply and demand, and form close loop. No diagonal adding or deducting since the supplies values located horizontally and demand values located vertically.
- d) Compute the improvement index for the empty cell (within close loop) by using formula;

$$I_{ij} = \sum \pm 1 * C_{ij}.$$
- e) Repeat the procedure till the indices of all empty cells are computed.
- f) Identifying the best empty cell, which realises max. reduction in transportation cost per unit.

Example: after developing the initial table by using the North West Corner the improvement indices will be computed as following:

	S1		S2		Supply
F1	-1	5	+1	4	100
	100				
F2	+1	8	-1	4	300
	200		100		
Demand	300		200		

$$I_{F1S2} = +4 - 4 + 8 - 5 = +3$$



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	S1		S2		S3		Supply
F1	-1	5		4	+1	3	100
	100						
F2	+1	8	-1	4		3	300
	200		100				
F3		9	+1	7	-1	5	300
			100		200		
Demand	300		200		200		

$$I_{F1S3} = +3 - 5 + 7 - 4 + 8 - 5 = +4$$

	S2		S3		Supply
F2	-1	4	+1	3	300
	100				
F3	+1	7	-1	5	300
	100		200		
Demand	200		200		

$$I_{F2S3} = +3 - 5 + 7 - 4 = +1$$

	S1		S2		Supply
F2	-1	8	+1	4	300
	200		100		
F3	+1	9	-1	7	300
			100		
Demand	300		200		

$$I_{F3S1} = +9 - 7 + 4 - 8 = -2 \dots \dots \text{Best cell.}$$

5.5.1.2 How to identify the amount will be transferred via best cell

- a) Return back to the close loop of the best cell.
- b) Identifying the cells within the close loop with negative signs.
- c) Identifying the minimum transferred quantity via negative sign cells. This quantity will be transferred via the best cell.
- d) Modify the close loop of the best cell by add and deduct the transferred amount via the empty cells.
- e) The cells out of close loop will be maintained the same. The result is the modified table or the next transportation table.

Example:

	S1		S2		Supply
F2	-1	8	+1	4	300
	200		100		
F3	+1	9	-1	7	300
			100		
Demand	300		200		

Above is the close loop of the best cell.

The negative sign cells are F2S1 and F3S2.

The min. Quantity transferred via negative sign cells us 100.

100 units will be transferred via the best cell.

So the modified loop is as follows.

	S1		S2		Supply
F2	-1	8	+1	4	300
	200	$200-100=100$	$100+100=200$		
F3	+1	9	-1	7	300
	$0+100=100$		$100-100=0$		
Demand	300		200		

The following is the next transportation table

- Total reduction in transportation cost= $100 * - 2 = - 200$ J.D.
- Total transportation cost as a result of improvement = $4200 - 200 = 4000$ J.D.

	S1	S2	S3	Supply
F1	5 100	4	3	100
F2	8 100	4 200	3	300
F3	9 100	7	5 200	300
Demand	300	200	200	700

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Second iteration

- a) Check number of occupied cells = 5
- b) Compute improvement indices
 - $I_{F1S2} = + 4 - 4 + 8 - 5 = + 3$
 - $I_{F1S3} = + 3 - 5 + 9 - 5 = + 2$
 - $I_{F2S3} = + 3 - 5 + 9 - 8 = - 1 \dots \dots \dots$ Best cell.
 - $I_{F3S2} = + 7 - 9 + 8 - 4 = + 2$
- c) The amount will be transferred via best cell is 100 units
- d) The modified close loop is as following

	S1		S2		S3		Supply
F2	-1	8		4	+1	3	300
	100-100=0		200		0+100=100		
F3	+1	9		7	-1	5	300
	100+100=200				200-100=100		
Demand	300		200		200		700
							700

- e) The next transportation table (third table)
 - Total reduction in transportation cost = $-1 \times 100 = -100$ J.D.
 - Total transportation cost = $4000 - 100 = 3900$ J.D.

	S1		S2		S3		Supply
F1	100	5		4		3	100
F2		8	200	4	100	3	300
F3	200	9		7	100	5	300
Demand	300		200		200		700
							700

Third iteration

- a) Check number of occupied cells = 5.
- b) Compute improvement indices

$$I_{F1S2} = +4 - 4 + 3 - 5 + 9 - 5 = +2$$

$$I_{F1S3} = +3 - 5 + 9 - 5 = +2$$

$$I_{F2S1} = +8 - 3 + 5 - 9 = +1$$

$$I_{F3S2} = +7 - 5 + 3 - 4 = +1$$

No best cell; since all indices are positives, no any negative index. So the third table is the optimal one.

Note: the third table is the same as the initial table developed by Vogel's method. So Vogel's method helps in reaching optimal solution for 80% of cases.

5.5.2 SECOND IMPROVEMENT METHOD: MODIFIED DISTRIBUTION METHOD (MODI)

According to this method the Transportation cost is distributed between origins and destinations.

5.5.2.1 How to compute the improvement indices

- a) Check the number of occupied cells. Same as previous.
- b) Assign R_i for origin and K_j for destinations. R_i represents the share of origin in transportation cost, and K_j represents the share of destination in transportation cost.
- c) Compute the values of R_i and K_j . The following equation us used

$$C_{ij} = R_i + K_j \dots \dots \dots (1)$$

C_{ij} : transportation cost per unit for occupied cell (ij).

R_i : share of origin (i) in transportation cost

K_j : share of destination in transportation cost.

- d) Compute the improvement indices for empty cells by using the following equation

$$I_{ij} = C_{ij} - R_i - K_j \dots \dots \dots (2)$$

C_{ij} : transportation cost per unit for the empty cell (ij)

R_i : share of origin (i) in transportation cost...computed in equation (1)

K_j : share of destination in transportation cost...computed in equation (1).

Example

		K1=5		K2=1		K3=-1		
		S1		S2		S3		Supply
R1=1	F1	100	5		4		3	100
R2=3	F2	200	8	100	4		3	300
R3=6	F3		9	100	7	200	5	300
		300		200		200		700
								700

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- Compute the values of R_i and K_j .

$$R_1 + K_1 = 5 \dots (1) \quad R_2 + K_1 = 8 \dots (2) \quad R_2 + K_2 = 4 \dots (3) \quad R_3 + K_2 = 7 \dots (4)$$

$$R_3 + K_3 = 5 \dots (5)$$

$$\text{Assume } R_1 = 0 \dots K_1 = 5 \dots R_2 = 3 \dots K_2 = 1 \dots R_3 = 6 \dots K_3 = -1.$$

- Compute the improvement indices

$$I_{F_1S_2} = 4 - 0 - 1 = +3, \quad I_{F_1S_3} = 3 - 0 - -1 = +4, \quad I_{F_2S_3} = 3 - 3 - -1 = +1$$

$$I_{F_3S_1} = 9 - 6 - 5 = -2 \dots \text{Best cell.}$$

5.5.2.2 How to identify the amount will be transferred via best cell

- Return back to the close loop of the best cell.
- Identifying the cells within the close loop with negative signs.
- Identifying the minimum quantity was transferred via negative sign cells. This quantity will be transferred via the best cell.
- Modify the close loop of the best cell by add and deduct the transferred amount via the empty cell.
- The remaining cells out of close loop will be maintained the same. The result is the modified table or the next transportation table.

Example:

	S1		S2		Supply
F2	-1	8	+1	4	300
	200		100		
F3	+1	9	-1	7	300
			100		
Demand	300		200		

Above is the close loop of the best cell.

The negative sign cells are F2S1 and F3S2.

The min. Quantity transferred via negative sign cells us 100.

100 units will be transferred via the best cell.

So the modified loop is as follows.

	S1		S2		Supply
F2	-1	8	+1	4	300
	200-100=100		100+100=200		
F3	+1	9	-1	7	300
	0+100=100		100-100=0		
Demand	300		200		700
					700

The following is the next transportation table

- Total reduction in transportation cost= $100 * - 2 = - 200$ J.D.
- Total transportation cost as a result of improvement = $4200 - 200 = 4000$ J.D.

	S1	S2	S3	Supply		
F1	100	5	4	3	100	
F2	100	8	200	4	3	300
F3	100	9	7	200	5	300
Demand	300	200	200		700	
					700	

5.6 SPECIAL CASES IN TRANSPORTATION

5.6.1 UNBALANCED TRANSPORTATION PROBLEM

It is a situation when total supply not equal total demand.

Procedure

1. Find the difference between supply and demand.
2. Add the difference as a dummy origin or destination for the lowest value.
3. The Transportation cost of dummy routes is zero.

Example

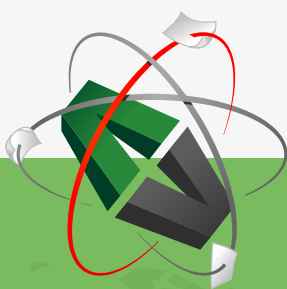
	S1	S2	S3	Supply
F1	5	4	3	100
F2	8	4	3	200
F3	9	7	5	300
Demand	300	200	200	600
				700

The previous problem is unbalanced. Demand > supply.

Difference between supply and demand is 100.

Add a dummy supply by 100.

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The following is balanced problem.

	S1	S2	S3	Supply
F1	5	4	3	100
F2	8	4	3	200
F3	9	7	5	300
Dummy F	0	0	0	100
Demand	300	200	200	600 700

5.6.2 DEGENERACY

- It is a special case in transportation occurs when the number of occupied cells is less than number of origin + number of destinations – 1.
- This case could occur after developing the initial table.
- This case could occur after improvement.

5.6.2.1 Degeneracy after developing initial table

This case could be solved by completing the ladder shape if North West corner method is adopted. The lowest cost cell that complete the ladder form will be assumed as occupied. If Vogel's or least cost method is adopted; the lowest empty cost cell will be assumed as occupied.

Example:

In the following example cell F1S2 is assumed as occupied cell since it completes the ladder form and realises minimum cost per unit.

	S1	S2	S3	Supply
F1	100	Occupied		100
F2		100		100
F3		200	100	300
Demand	100	300	100	500
				500

5.6.2.2 Degeneracy after improvement

Example

	S1	S2	S3	Supply
F1	100			100
F2	100	100		200
F3		100	200	300
Demand	200	200	200	600
				600

The best empty cell is F3S1

The amount will be transferred via best cell is 100

So the next table will be as following

	S1	S2	S3	Supply
F1	5 100	4	3	100
F2	8 Occupied	4 200	3	200
F3	9 100	7	5 200	300
Demand	200	200	200	600 600

The number of occupied cells in second table is 4 which less than 5. To cope with problem assume the negative sign cell which has lowest cost within the close loop of the best cell as an occupied. So cell F2S1 will be assumed as an occupied.

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5.7 REVIEW QUESTIONS

1. The improvement indices of empty cells of transportation problem are presented in the following table. Please answer the following questions

Empty cells	Improvement index
O1D2	+3
O1D3	0
O2D1	+5
O2D3	-6
O3D1	-7

a) What is the best empty cell? Please justify your answer.

b) Please explain the improvement index of

O1D2:.....

O1D3:.....

O3D1:.....

c) If the close loop of the best empty cell is as presented in the following matrix, please answer the next questions

	10		5
80		100	
	6		8
		50	

The amount will be transferred via the best empty cell is:.....

The total transportation cost will be reduced by:.....

If the initial transportation cost is 2100 J.D. the total transportation cost will be.....

2. The following is the transportation table for a particular problem. Please answer the following questions

	D1	D2	D3	D4	Supply
O1	8	4	6	4	200
O2	5	3	5	5	100
O3	4	2	2	9	200
O4	2	5	1	7	200
Demand	100	200	300	200	

a) Please develop the initial transportation table by using north west corner method

b) Please compute the improvement indices for the following empty cells O2D1, O3D4, O3D2. (3 Marks)

6 DECISION MAKING MODELS

6.1 LEARNING OUTCOMES

1. Develop the decision making table and tree.
2. Applying the decision making principles under uncertain environment.
3. Applying the decision making principles under risky environment.

6.2 COMPONENTS OF DECISION

- a) Decision alternatives.

All reachable alternatives should be listed.

Do nothing alternative should be listed too.

- b) Decision environment (states of nature).

The environment could be internal or external, which includes all forces or factors.

The factors as customer should be identified and its related indicators as size of demand should be identified too.

The behaviour of data of environmental indicator should be stationary.




So the indicators could be classified to different levels.

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The environment according to uncertainty could be classified to: uncertain, risky and certain.

Risky environment; the decision maker has a past experience in dealing with such environment so he can assign probabilities of its occurrence.

Uncertain environment; the decision maker has not a past experience to deal with such environment so he is unable to assign probabilities of its occurrence.

Certain environment; whether the environment is stable, or the changing of environment has not an impact on the decision outcomes.

The majority of decisions are Risky.

c) Decision outcome (payoff).

It is a performance indicators as profit, cost, market share, customer satisfaction, customer retention...etc.

Under each state of nature corresponds each decision alternative there is an outcome.

6.3 PRINCIPLES OF MAKING UNCERTAIN DECISION

Example: the chief officers of Delta airlines after 11/9 decided to make strategic decision, the following table shows the alternatives and payoffs. Could you please identify the best alternative?

Decision Alternatives	Level of demand			optimistic Max.	Pessimistic Min	Wp realism	Equally likely
	Low	Moderate	High				
Merger	(50,000)	50,000	100,000	Max. 100,000	Max. (50,000)	Max. 70,000	Max. 33,333
Bankruptcy	(100,000)	(50,000)	50,000	50,000	(100,000)	20,000	(33,333)
Closing down	(100,000)	(100,000)	(100,000)	(100,000)	(100,000)	(100,000)	(100,000)
Do nothing	(200,000)	(150,000)	(50,000)	(50,000)	(200,000)	(80,000)	(133,333)

6.3.1 PESSIMISTIC PRINCIPLE (MAX. MAX)

- a) Identifying the maximum payoff corresponds each decision.
- b) Identifying the max. of max. payoff, the corresponding decision is the best.

6.3.2 PESSIMISTIC PRINCIPLE (MAX. MIN.)

- a) Identifying the minimum payoff corresponded each decision.
- b) Identifying the max. of min. payoffs. the corresponding decision is the best.

6.3.3 HURWICZ (REALISM CRITERION)

- a) Identifying the max. payoff corresponds each decision.
- b) Identifying min. payoff corresponds each decision.
- c) Identifying the probability of optimistic and pessimistic.
- d) Compute weighted Payoff (Wp) by multiplying the max. payoff by the probability of optimistic and min. payoff by the probability of pessimistic and sum the results.
- e) Identifying the max. (Wp), the corresponding decision is the best.

6.3.4 EQUALLY LIKELY (LAPLACE)

- a) Compute average payoff for each alternative.
- b) Identifying the max. averaged payoff, corresponding decision is the best.

6.3.5 MIN. MAX REGRET

- a) Developing opportunity the loss table.
- b) Identifying max Payoff under each state of nature.
- c) deduct the Payoff of each alternative under each state of nature from the max. the result is the opportunity loss.
- d) The following table is the opportunity loss table.
- e) Identifying max. opportunity loss of each alternative.
- f) Identifying min. opportunity loss, corresponding decision is the best.

Decision Alternatives	Level of demand			Max.
	Low	Moderate	High	
Merger	0	0	0	Min. 0
Bankruptcy	50,000	100,000	50,000	100,000
Closing down	50,000	150,000	200,000	200,000
Do nothing	150,000	200,000	250,000	250,000

6.4 DECISION MAKING UNDER RISKY ENVIRONMENT PRINCIPLES

The decision maker has a past experience so he is able to assign probabilities for the states of nature occurrence.

6.4.1 EXPECTED MONETARY VALUE (EMV)

- a) Compute the weighted payoff by multiplying the payoff of each state nature by the probability of occurrence, then, sum the result. The result is called EMV.
- b) Identifying the maximum EMV; the corresponding decision is the best.

Example; the operations manager of hospital decided to identify the best capacity decision; the following is the payoff table. Could you please identify the best alternative by using EMV?.

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Alternatives	Level of demand			EMV
	Low	Moderate	High	
New building	353500	736000	1456000	$353500 \times 0.4 + 736000 \times 0.3 + 1456000 \times 0.3 = 799000$ maximum EMV
Extend building	176750	365000	728000	$176750 \times 0.4 + 365000 \times 0.3 + 728000 \times 0.3 = 398500$
Do nothing	88375	184000	364000	$88375 \times 0.4 + 184000 \times 0.3 + 364000 \times 0.3 = 199750$
Probability	0.4	0.3	0.3	

6.4.2 EXPECTED OPPORTUNITY LOSS (EOL)

- a) Developing the opportunity loss table.
- b) Compute the weighted opportunity loss for each alternative by multiplying the opportunity loss of each state nature by the probability of occurrence, then, sum the results. The result is called EOL.
- c) Identifying the minimum EOL; the corresponding decision is the best.

Example; the operations manager of hospital decided to identify the best capacity decision; the following is the payoff table. Could you please identify the best alternative by using EOL?.

Alternatives	payoff table			opportunity loss table			
	Level of demand			Level of demand			
	Low	Moderate	High	Low	Moderate	High	EOL
New building	353500	736000	1456000	0	0	0	0 Min. EOL.
Extend building	176750	365000	728000	176750	371000	728000	400400
Do nothing	88375	184000	364000	265125	552000	1092000	599250
Probability	0.4	0.3	0.3	0.4	0.3	0.3	

6.5 HOW TO DEVELOP THE PAYOFF TABLE

1. Identifying the decision alternatives; for example the case of the capacity planning, accordingly the capacity alternatives are;
 - a) New building
 - b) Extend building
 - c) Do nothing
2. Identifying the environmental factor indicator
The environmental factor indicator which determine the capacity is the size of demand
3. Identifying the levels of environmental factor indicator and its scale
The scale could be identified through revising the historical data of demand

Level of demand	Scale
Low	Demand ≤ 2500
Moderate	$2500 < \text{demand} \leq 7000$
High	Demand > 7000

4. Compute the probability of demand's level; to do that historical data will be retrieved as presented below.
 - a) Classifying the demand to levels; low moderate and high as presented in the table.

Year	Demand	Level
2005	1750	Low
2006	3100	Moderate
2007	1850	Low
2008	3600	Moderate
2009	8100	High
2010	2100	Low
2011	5600	Moderate
2012	8600	High
2013	7600	High
2014	2200	Low

- b) Find the frequency of each level of demand, then, divide the frequency of each level by the total frequencies, as presented in the following table, the result is the probability of each level of demand occurrence.

Level of demand	Frequency	Probability
Low	4	$4/10 = 0.4$
Moderate	3	$3/10 = 0.3$
High	3	$3/10 = 0.3$
Total	10	1.00

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5. Forecasted each level of demand; the forecasted value could be computed by using different techniques. In this example the average will be used for this purpose.

Year	Low demand values
2005	1750
2007	1850
2010	2100
2014	2200
Forecasted	$(1750+1850+2100+2200)/4= 1975$

year	Moderate demand values
2006	3100
2008	3600
2011	5600
Forecasted	$(3100+3600+5600)/3= 4100$

Year	High demand values
2009	8100
2012	8600
2013	7600
Forecasted	$(8100+8600+7600)/3= 8100$

6. Compute the payoff of each alternative under each state of nature
- In this example the payoff is the profit.
 - The profit formula is: Profit= Price/unit * quantity – fixed cost – variable cost/unit * quantity.
 - So for each alternative the price/unit, fixed cost and variable cost/unit should be known. These facts are summarized in the following table.

Alternatives	Price/unit	Fixed cost	Variable cost/unit
New building	200	2000	20
Extend building	100	1000	10
Do nothing	50	500	5

- Payoffs under low demand (low demand = 1975).
 New building= $200 \times 1975 - 2000 - 20 \times 1975 = 353500$ J.D.
 Extend building= $100 \times 1975 - 1000 - 10 \times 1975 = 176750$ J.D.
 Do nothing= $50 \times 1975 - 500 - 5 \times 1975 = 88375$ J.D.
- Payoffs under moderate demand (moderate demand = 4100).
 New building= $200 \times 4100 - 2000 - 20 \times 4100 = 736000$ J.D.
 Extend building= $100 \times 4100 - 1000 - 10 \times 4100 = 368000$ J.D.
 Do nothing= $50 \times 4100 - 500 - 5 \times 4100 = 184000$ J.D.
- Payoffs under high demand (high demand = 8100).
 New building= $200 \times 8100 - 2000 - 20 \times 8100 = 1456000$ J.D.
 Extend building= $100 \times 8100 - 1000 - 10 \times 8100 = 728000$ J.D.
 Do nothing= $50 \times 8100 - 500 - 5 \times 8100 = 364000$ J.D.

7. Combine the result in a table called payoff table

Alternatives	Level of demand		
	Low	Moderate	High
New building	353500	736000	1456000
Extend building	176750	365000	728000
Do nothing	88375	184000	364000
Probability	0.4	0.3	0.3

6.6 DECISION TREE

- It is a technique used for making a risky decision when the decisions are multi – layers or when states of nature are multi – layers or both.
- The arrows and nodes are used to represent the decision alternative and states of nature.
- The decision category is represented by square nodes.
- Decision alternatives are presented by the circle nodes.
- The branches (arrows) used to represents decision alternatives or states of nature.
- At the end of each tree's branches, the payoff is represented.
- Over or under each arrow of each state of nature the probability is represented.
- Over or under each decision alternative arrow the EMV is represented.
- Two dashed diagonally lines are drawn on the arrow of best decision alternatives.



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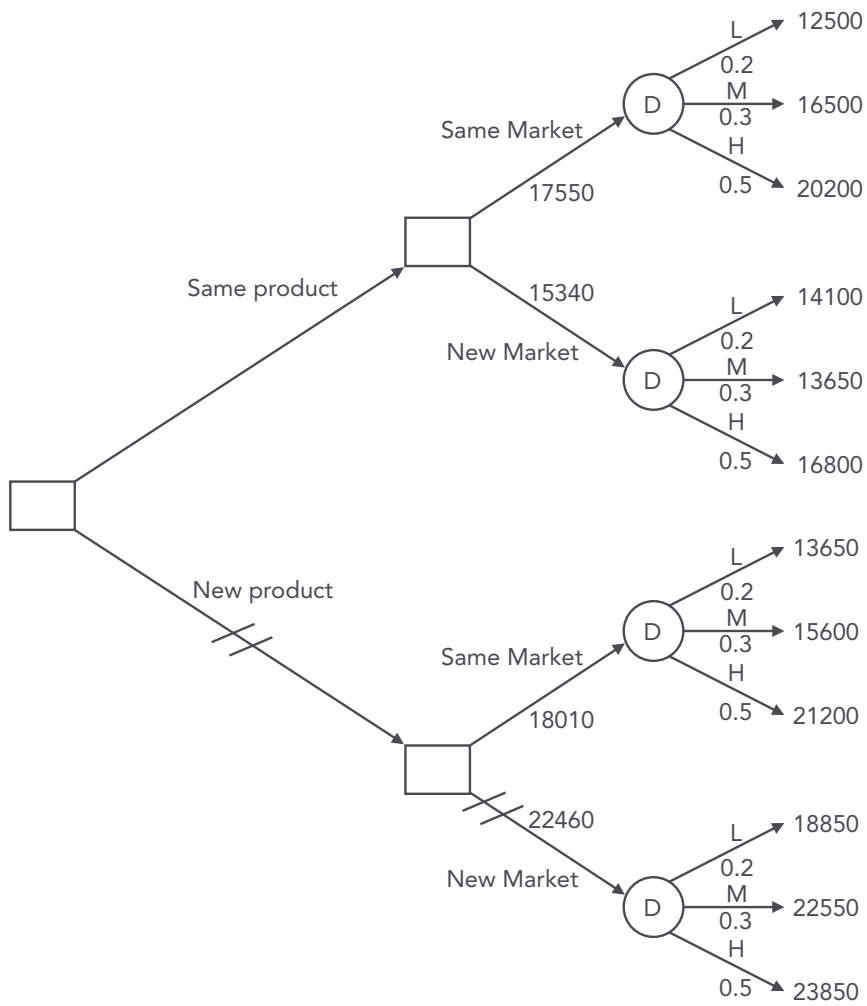
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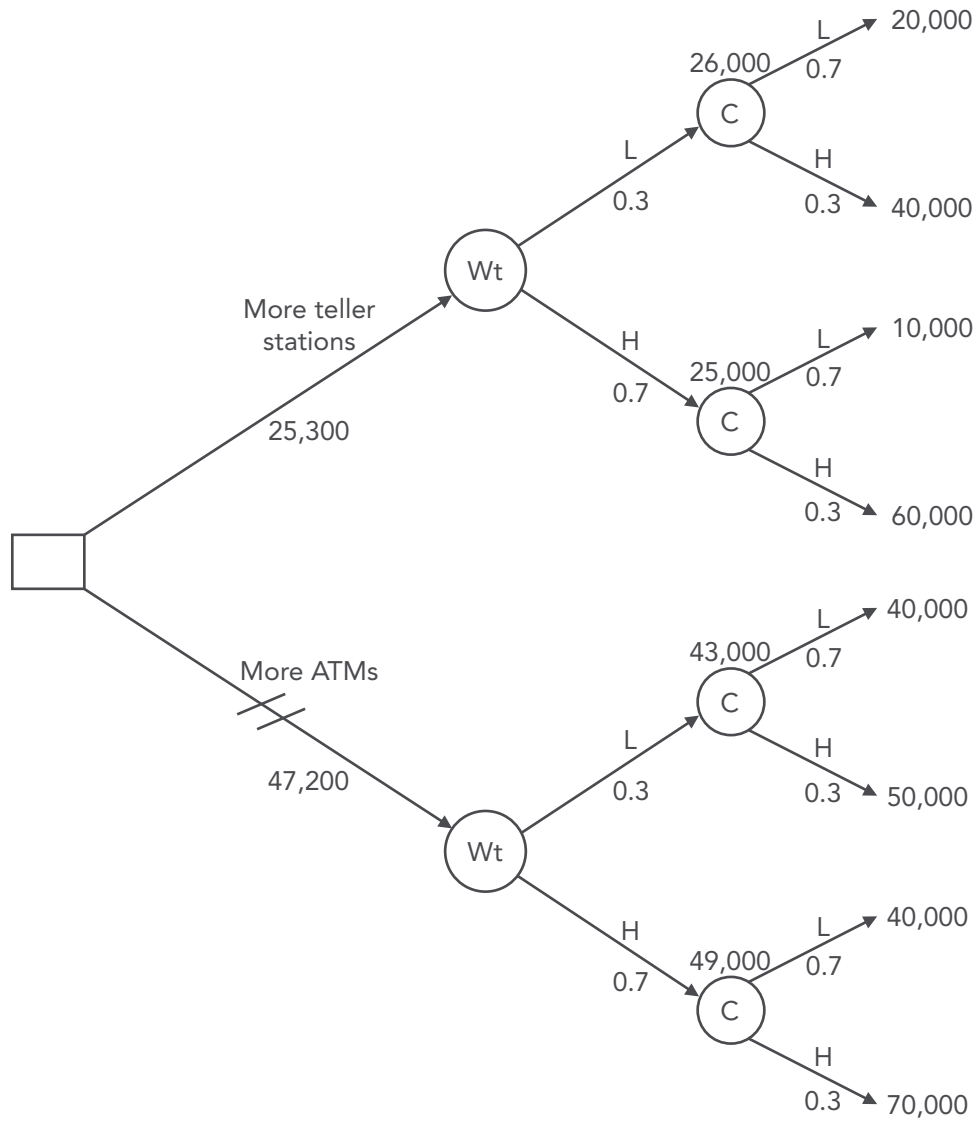
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Example: multi-decision layers



The best decision is developing new product for new market and this decision will realize an EMV by 22460 J.D.

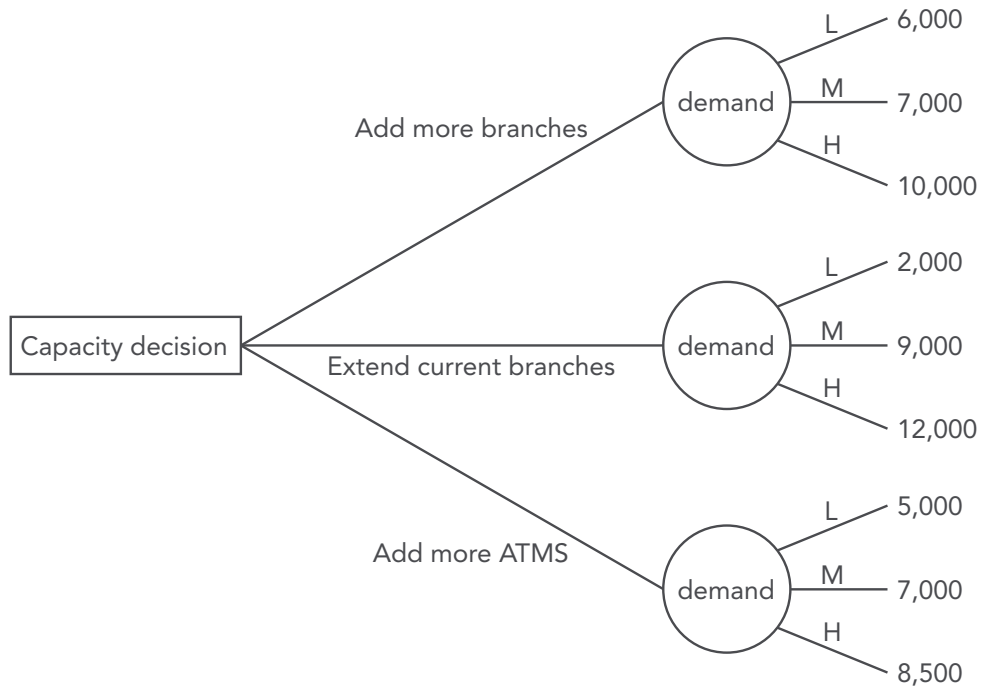
Example: multi-states of nature layers



The best decision is add more ATMs and this decision will realize an EMV by 47,200 J.D.

6.7 REVIEW QUESTIONS

1. The following is the capacity decision tree of a bank's branches. Please answer the following questions.



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a) Please identify the probability of each level of demand if the historical demand values are as presented in the following table. And the scales are as presented in the next table.

year	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013
demand	2000	4000	6000	3000	10,000	5000	9000	3000	4000	2000

Scale range	Level of demand
0-0.3	Low
>0.3-0.7	Moderate
>0.7-1.00	High

b) What is the best alternative according to EMV.?

2. The following is the strategic decision making table for a bank in USA after the financial crisis of 2008. Please answer the following questions

Decision alternative	Level of demand		
	low	Moderate	High
Bankruptcy	-100,000	-50,000	10,000
Merger	-50,000	50,000	70,000
Closing down	-100,000	-100,000	-100,000

a) What is the best alternative according to Minimax Regret principle?

b) What is the best alternative according to Maxmax?

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