Description of Motion in One Dimension

2
CHAPTER

AIEEE Syllabus

Frame of reference. Motion in a straight line: Position-time graph, speed and velocity. Uniform and non-uniform motion, average speed and instantaneous velocity, Uniformly accelerated motion, velocity-time, position-time graphs, relations for uniformly accelerated motion.

THIS CHAPTER COVERS :

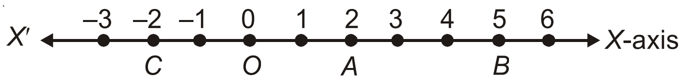
- Position
- Displacement and Velocity
- Distance and speed
- Acceleration
- Equations of Motion
- Graphs
- Motion under gravity
- Relative motion in one dimension

POSITION AND FRAME OF REFERENCE

Position of an object is specified with respect to a **reference frame.**

In a reference frame, an observer measures the position of the other object at any instant of time, with respect to a coordinate system chosen and fixed arbitrarily on the reference frame.

for example : The position of particle at *O*, *A*, *B* and *C* are Zero, +2, +5 and –2 respectively with respect to origin (*O*) of reference frame.



DISPLACEMENT AND VELOCITY

Displacement

1. The change in position of a body in a certain direction is known as displacement.
2. The distance between the initial and final position is known as **magnitude of displacement.**
3. Displacement of an object may be positive, negative or zero and it is independent of the path followed by the object.
4. Its SI unit is meter and dimensional formula is $[M^0L^1T^0]$.

Velocity

1. **Average velocity :** [<*v*>] :

   If Δx is displacement in time Δt, then average velocity in time interval Δt will be

   $$< v > = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}.$$

   Here x_f and x_i be the position of particle at time t_f and t_i ($t_f > t_i$) with respect to a given frame of reference.

2. **Instantaneous velocity (*v*) :** It is the velocity of particle at any instant of time

   Mathematically,

   $$v = \underset{\Delta t \to 0}{\text{Limit}} < v > = \underset{\Delta t \to 0}{\text{Limit}} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

DISTANCE AND SPEED

Distance

1. The total length of actual path traversed by the body between initial and final positions is called distance.
2. It has no direction and is always positive.
3. Distance covered by particle never decreases.
4. Its SI unit is meter (m) and dimensional formula is $[M^0L^1T^0]$.

Speed

1. **Average speed :** It is defined as distance travelled by particle per unit time in a given interval of time.

   If S is the distance travelled by particle in time interval *t*, then average speed in that time interval is $\frac{S}{t}$.

2. **Instantaneous speed :** The magnitude of instantaneous velocity at a given instant is called instantaneous speed at that instant.

ACCELERATION

Time rate of change of velocity is called **acceleration.**

1. Average acceleration : If Δv is change in velocity in time Δt, then average acceleration in time interval Δt is

   $$< a > = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}.$$

2. **Instantaneous acceleration :** The acceleration at any instant is called **instantaneous acceleration.** Mathematically

   $$a = \underset{\Delta t \to 0}{Limit} < a > = \underset{\Delta t \to 0}{Limit} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}.$$

Uniform and variable acceleration :

If the change in velocity of the particle is equal in equal intervals of time, then the acceleration of the body is said to be **uniform**. Neither direction, nor magnitude changes with respect to time.

If cnange in velocity is different in equal intervals of time, then the acceleration of the particle is known as **variable**. If either direction or magnitude or both magnitude and direction of acceleration changes with respect to time, then acceleration is **variable.**

EQUATIONS OF MOTION

General equations of motion :

$$v = \frac{dx}{dt} \Rightarrow dx = vdt \Rightarrow \int dx = \int vdt$$

$$a = \frac{dv}{dt} \Rightarrow dv = adt \Rightarrow \int dv = \int adt$$

$$a = \frac{vdv}{dx} \Rightarrow vdv = adx \Rightarrow \int vdv = \int adx$$

Equations of motion of a particle moving with uniform acceleration in straight line :

1. $v = u + at$

2. $S = ut + \frac{1}{2}at^2$

3. $v^2 = u^2 + 2aS$

4. $S_{n^{th}} = u + \frac{1}{2}a(2n - 1)$

5. $x = x_0 + ut + \frac{1}{2}at^2$

   Here

   u = velocity of particle at $t = 0$

   S = Displacement of particle between 0 to t

   $= x - x_0$ (x_0 = position of particle at $t = 0$, x = position of particle at time t)

   a = uniform acceleration

   v = velocity of particle at time t

   $S_{n^{th}}$ = Displacement of particle in n^{th} second

Average Speed and Velocity

1. If a body covers s_1 distance with speed v_1, s_2 with speed v_2, then its average speed is

   $$v_{av} = \frac{s_1 + s_2}{\frac{s_1}{v_1} + \frac{s_2}{v_2}} = \frac{\sum s}{\sum \frac{s}{v}}$$

2. If a body coves first half distance with speed v_1 and next half with speed v_2, then

   Average speed = $\frac{2v_1v_2}{v_1 + v_2}$ (Harmonic mean)

3. If a body travels with uniform speed v_1 for time t_1 and with uniform speed v_2 for time t_2, then average speed

   $$= \frac{v_1t_1 + v_2t_2}{t_1 + t_2} = \frac{\sum vt}{\sum t}.$$

   If $t_1 = t_2 = \frac{T}{2}$ then $v_{av} = \frac{v_1 + v_2}{2}$ [T = time of journey] (Arithmatic mean)

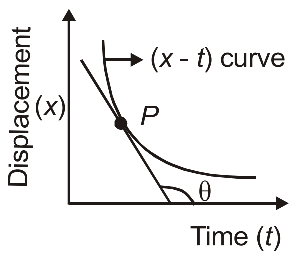
4. If body covers first one third with speed v_1, next one third with speed v_2 and remaining one third with speed v_3 then $v_{av} = \frac{3v_1v_2v_3}{v_1v_2 + v_2v_3 + v_3v_1}$.

5. If a body moves from one point (A) to another point (B) with speed v_1 and returns back (from B to A) with speed v_2 then average velocity is 0 but average speed = $\frac{2v_1v_2}{v_1 + v_2}$.

GRAPHS

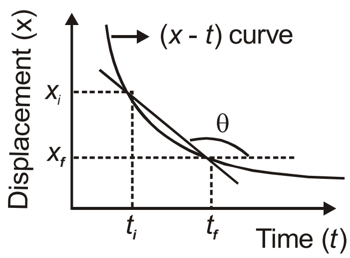
The important properties of various graphs are given below :

1. Slope of the tangent at a point on the displacement-time graph gives the instantaneous velocity at that point.



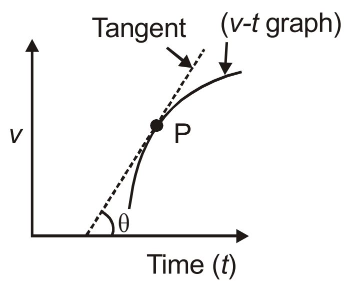
$$\tan\theta = \frac{dx}{dt} = v$$ (**Instantaneous Velocity** at point P)

2. Slope of the chord joining two points on the displacement-time graph gives the average velocity during the time interval between those points.



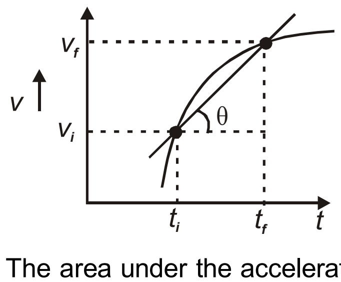
$$\tan\theta = \frac{x_f - x_i}{t_f - t_i} = V_{av}$$

3. Slope of the tangent at a point on the velocity-time graph gives the instantaneous acceleration at that point.



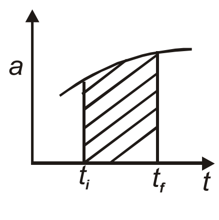
$$\tan\theta = \frac{dv}{dt} = a$$ (Instantaneous acceleration at P)

4. Slope of the chord joining two points on the velocity-time graph gives the average acceleration during the time interval between those points.



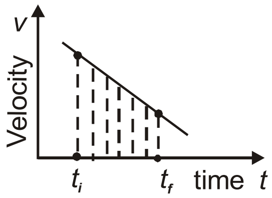
$$\tan\theta = \frac{v_f - v_i}{t_f - t_i} = a_{av}$$ = Average acceleration in time interval $t_f - t_i$

5. The area under the acceleration-time graph between t_i and t_f gives the change in velocity ($v_f - v_i$) between the two instants.



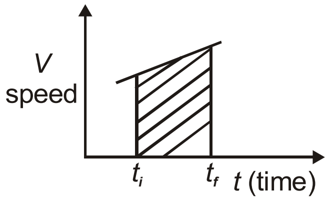
Shaded area = $v_f - v_i$ = change in velocity

6. The area under speed-time graph between t_i and t_f gives distance covered by particle in the interval $t_f - t_i$.



Shaded area = displacement in time $(t_f - t_i)$

7. The area under the velocity-time graph between t_i and t_f gives the displacement $(x_f - x_i)$ between the two instants.

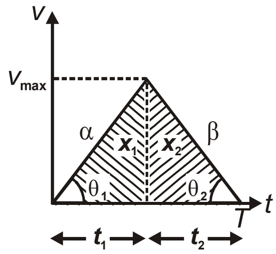


Shaded area = distance covered in time $(t_f - t_i)$

8. The displacement-time graph cannot take sharp turns because it gives two different velocities at that point.
9. The displacement-time graph cannot be symmetric about the time-axis because at an instant a particle cannot have two displacements, but the graph may be symmetric about the displacement-axis.
10. The distance-time graph is always an increasing curve for a moving body.
11. The displacement-time graph does not show the trajectory of the particle.

Applications

1. If a particle is moving with uniform acceleration and have velocity V_A at A and V_B at B, then velocity of particle midway on line AB is $V = \sqrt{\dfrac{V_A^2 + V_B^2}{2}}$.
2. If a body starts from rest with acceleration α and then retards to rest with retardation β, such that total time of journey is T, then



(a) Maximum velocity during the trip $v_{max.} = \left(\dfrac{\alpha\beta}{\alpha+\beta}\right).T$

(b) Length of the journey $L = \dfrac{1}{2}\left(\dfrac{\alpha\beta}{\alpha+\beta}\right)T^2$

(c) Average velocity of the trip = $\dfrac{v_{max.}}{2} = \dfrac{\alpha\beta T}{2(\alpha+\beta)}$ & (d) $\dfrac{x_1}{x_2} = \dfrac{\beta}{\alpha} = \dfrac{t_1}{t_2}$.

MOTION UNDER GRAVITY

Whenever a particle is thrown up or down or released from a height, it falls freely under the effect of gravitational force of earth.

The equations of motion :

1. $v = u + gt$
2. $h = h_0 + ut + \dfrac{1}{2}gt^2$ or $h - h_0 = s = ut + \dfrac{1}{2}gt^2$
3. $v^2 = u^2 + 2g(h - h_0)$ or $v^2 = u^2 + 2gs$
4. $h_{n^{th}} = u + \dfrac{g}{2}(2n-1)$

where h = vertical displacement, $h_{n^{th}}$ = vertical displacement in n^{th} second

Following are the important cases of interest.

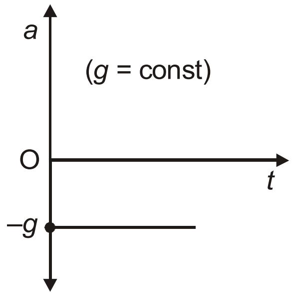
1. A particle is projected from ground with velocity u in vertically upward direction then

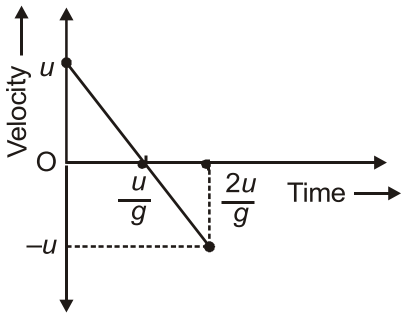
   (a) Time of ascent = Time of descent = $\frac{\text{Time of flight}}{2} = \frac{T}{2} = \frac{u}{8}$

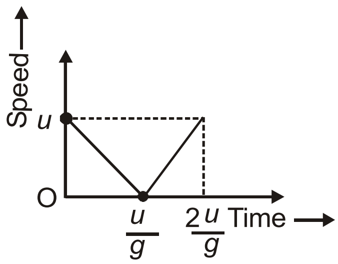
   (b) Maximum height attained = $\frac{u^2}{2g}$

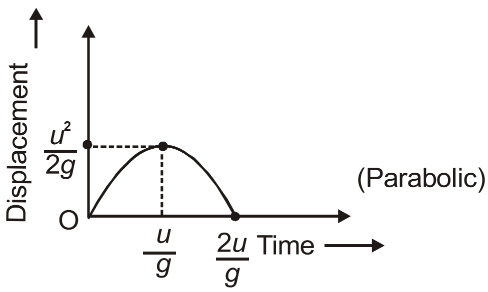
   (c) Speed of particle when it hits the ground = u

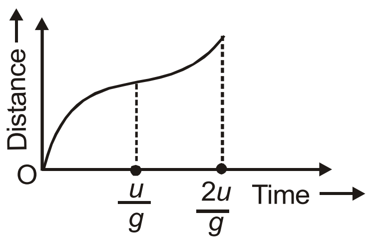
   (d) Graphs











   (e) Displacement of particle in complete journey = zero ⇒ average velocity $v_{av} = 0$

   (f) Distance covered by particle in complete journey = $\frac{u^2}{g}$

   ⇒ Average speed in complete journey = $\frac{u}{2}$

2. A body is thrown upward such that it takes t seconds to reach its highest point.

   (a) Distance travelled in $(t)^{th}$ second = distance travelled in $(t + 1)^{th}$ second.

   (b) Distance travelled in $(t – 1)^{th}$ second = distance travelled in $(t + 2)^{th}$ second.

   (c) Distance travelled in $(t – r)^{th}$ second = distance travelled in $(t + r + 1)^{th}$ second.

3. A body is projected upward from certain height h with initial speed u.

   (a) Its speed when it acquires the at same level is u.

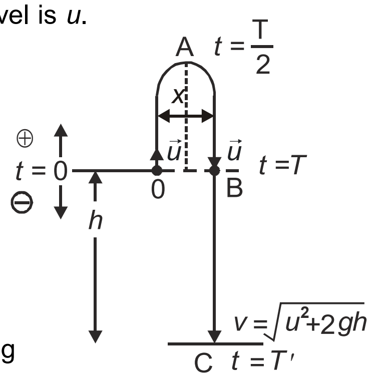
   (b) Its speed at the ground level is

   $$v = \sqrt{u^2 + 2gh}$$

   (c) The time require to attain same level is

   $$T = \frac{2u}{g}$$

   (d) Total time of flight (T) is obtained by solving



Here $x = 0$ particle follows same path during ascent and descent

$$-h = +uT' + \frac{1}{2}gT'^2 \text{ or } T' = \frac{T + \sqrt{T^2 + \frac{8h}{g}}}{2}, \text{ where } T = \frac{2u}{g}$$

Some Important Points :

1. During free fall distance increases by equal amounts *i.e., g* during 1st , 2nd, 3rd seconds of fall, *i.e.,* 4.9m, 14.7m, 24.5m are the distances travelled during 1st, 2nd and 3rd seconds respectively.
2. From the top of a tower a body is projected upward with a certain speed, 2nd body is thrown downward with same speed and 3rd is let to fall freely from same point then

   $$t_3 = \sqrt{t_1 t_2}$$

   where, t_1 = Time of flight of body projected upward

   t_2 = Time of flight of body thrown downward

   t_3 = Time of flight of body dropped.
3. If a body falls freely from a height h on a sandy surface and it buries into sand upto a depth of x, then the retardation with which body travels in the sand is

   $$a = \frac{gh}{x}$$
4. If u and v are velocity of particle at $t = t_1$ and $t = t_2$, which is moving with uniform acceleration, then average velocity of particle during time interval ($t_2 - t_1$) is $V_{av} = \frac{u+v}{2}$.
5. For a body starting from rest and moving with uniform acceleration, the ratio of distances covered in 1s, 2s, 3s, *etc.* is $1^2 : 2^2 : 3^2$ etc., *i.e.* 1 : 4 : 9 etc.
6. A body starting from rest and moving with uniform acceleration has distances covered by it in 1st, 2nd and 3rd seconds in the ratio 1 : 3 : 5 etc. i.e., odd numbers only.
7. A body moving with a velocity v is stopped by application of brakes after covering a distance s. If the same body moves with a velocity nv, it stops after covering a distance n^2s by the application of same brake force.
8. In the absence of air resistance, the velocity of projection is equal to the velocity with which the body strikes the ground.
9. In case of air resistance, the time of ascent is less than time of descent for a body projected vertically upward.
10. For a body projected vertically upwards, the magnitude of velocity at any given point on the path is same whether the body is moving in upwards or downward direction.

RELATIVE MOTION IN ONE DIMENSION

1. If two bodies *A* and *B* are moving in straight line same direction with velocity V_A and V_B, then relative velocity of *A* with respect to *B* is $v_{AB} = v_A - v_B$. Similarly $v_{BA} = v_B - v_A$

2. If two bodies *A* and *B* are moving in straight line in opposite direction then

$v_{AB} = v_A + v_B$ (towards *B*)

$v_{BA} = v_B + v_A$ (towards *A*)

$v_{AB} = -v_{BA}$

Same concept is used for acceleration also.

3. If two cars *A* and *B* are moving in same direction with velocity v_A and v_B ($v_A > v_B$) when *A* is behind *B* at a distance *d*, driver in car *A* applies brake which causes retardation *a* in car A, then minimum value of *d* to avoid collision is $\frac{(v_A - v_B)^2}{2a}$ *i.e.*, $d > \frac{(v_A - v_B)^2}{2a}$.

4. A particle is dropped and another particle is thrown downward with initial velocity *u*, then
   (a) Relative acceleration is always zero
   (b) Relative velocity is always *u*.
   (c) Time at which their separation is *x* is $\frac{x}{u}$.

5. Two bodies are thrown upwards with same initial velocity with time gap τ. They will meet after a time *t* from projection of first body.

$$t = \frac{\tau}{2} + \frac{u}{g}$$

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